

Chapter 1: Getting Started

$$E[X] = \mu = \frac{1}{n} \sum_{i=0}^{n-1} x_i$$

$$\langle \vec{v}, \vec{w} \rangle = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \quad \alpha: \vec{v} \rightarrow \langle \vec{v}, \vec{w} \rangle$$

$$p(X, Y) = p(X \cap Y) = p(X) \cdot p(Y)$$

$$p(Y|X) = p(X|Y) \cdot p(Y) / p(X)$$

$$p(Y|X) = p(Y, X) / p(X)$$

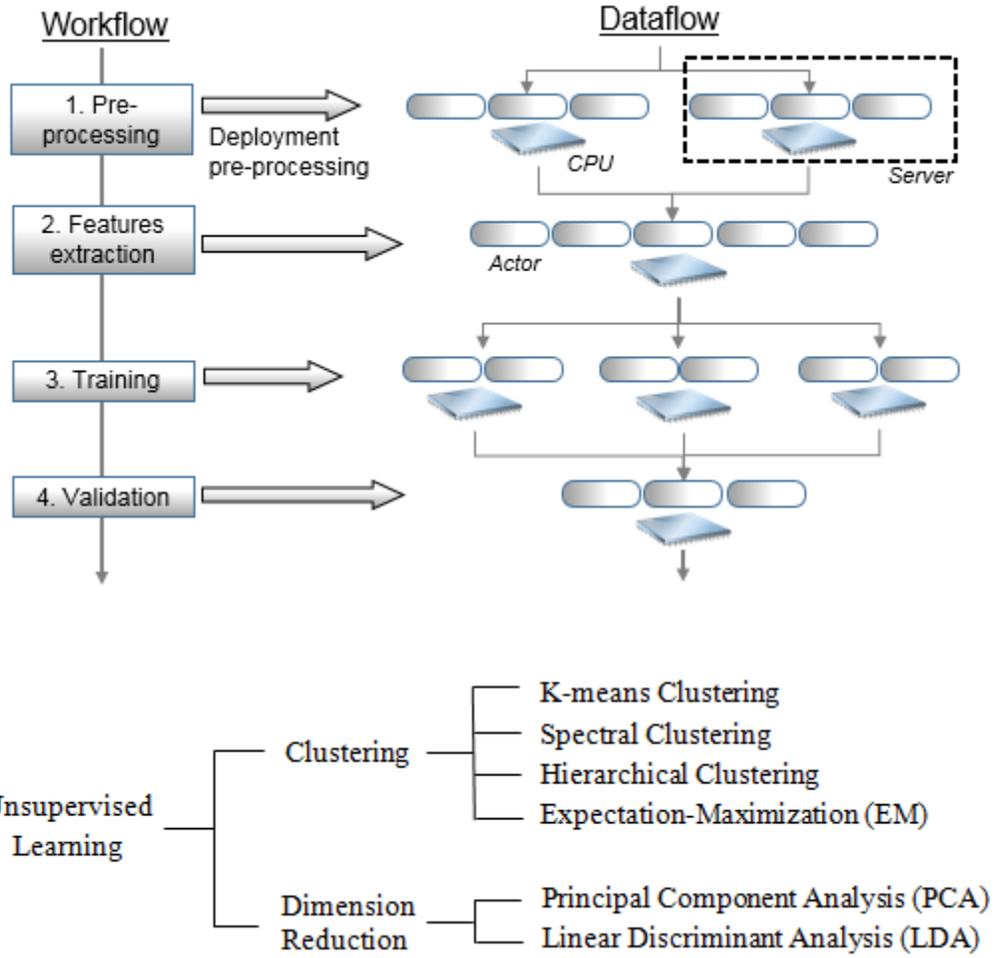
$$y_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}$$

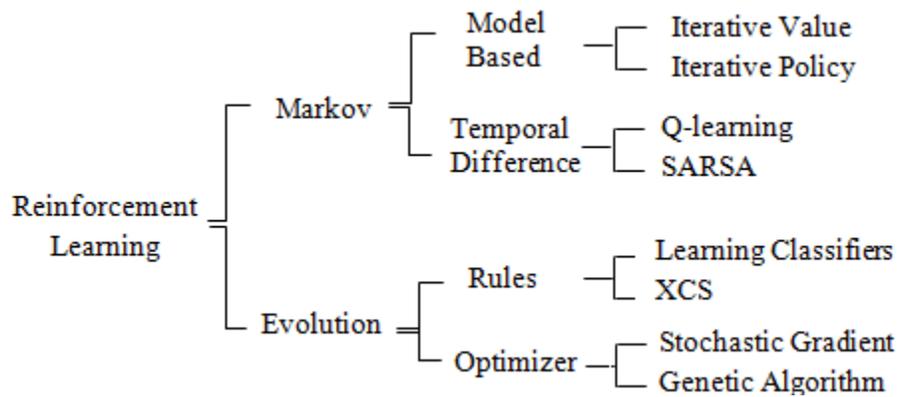
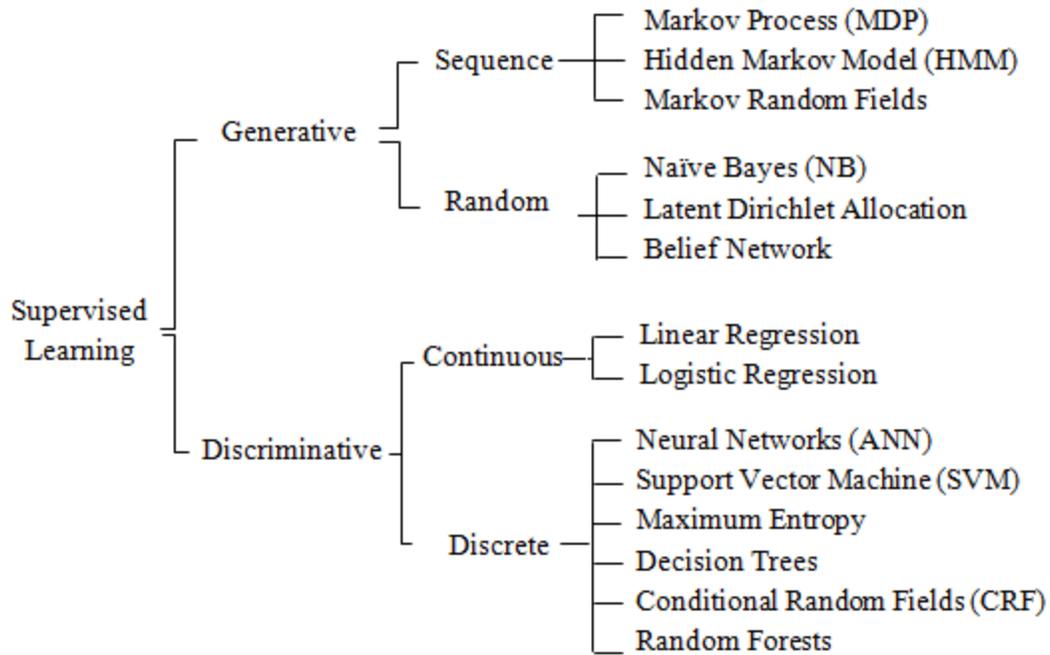
$$y_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} (h - l) + l$$

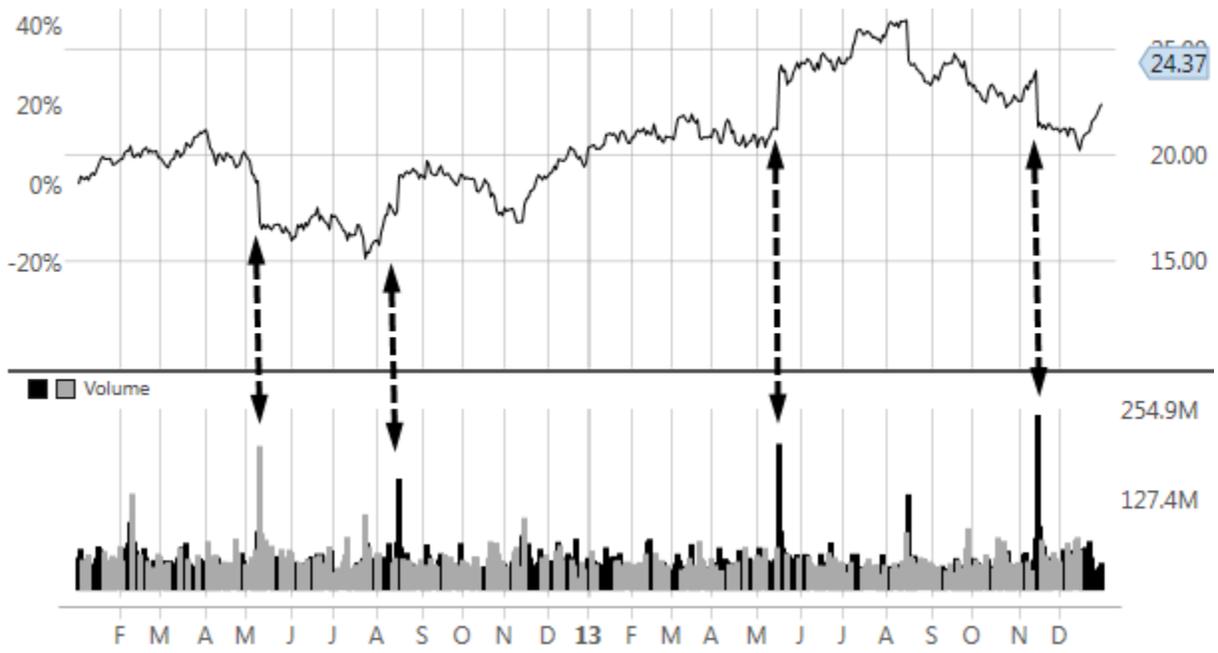
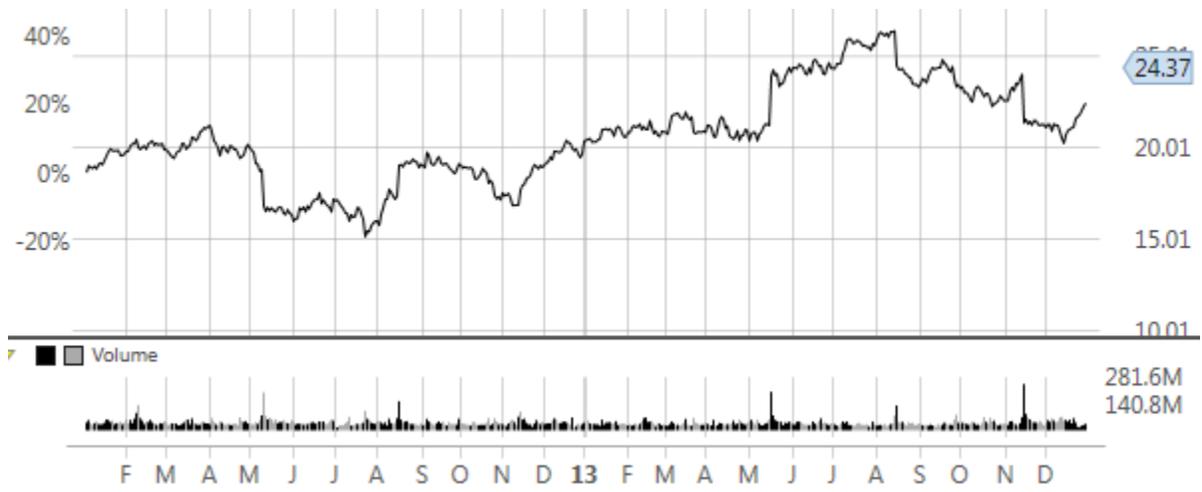
$$f(x|w) = w_0 + \sum_{i=1}^{N-1} x_i w_i \quad l(x|w) = \frac{1}{1 + e^{-f(x|w)}}$$

$$\mathcal{L}(x|w) = \frac{1}{N} \sum_{n=0}^{N-1} \log(1 + e^{-y_n w^T x})$$

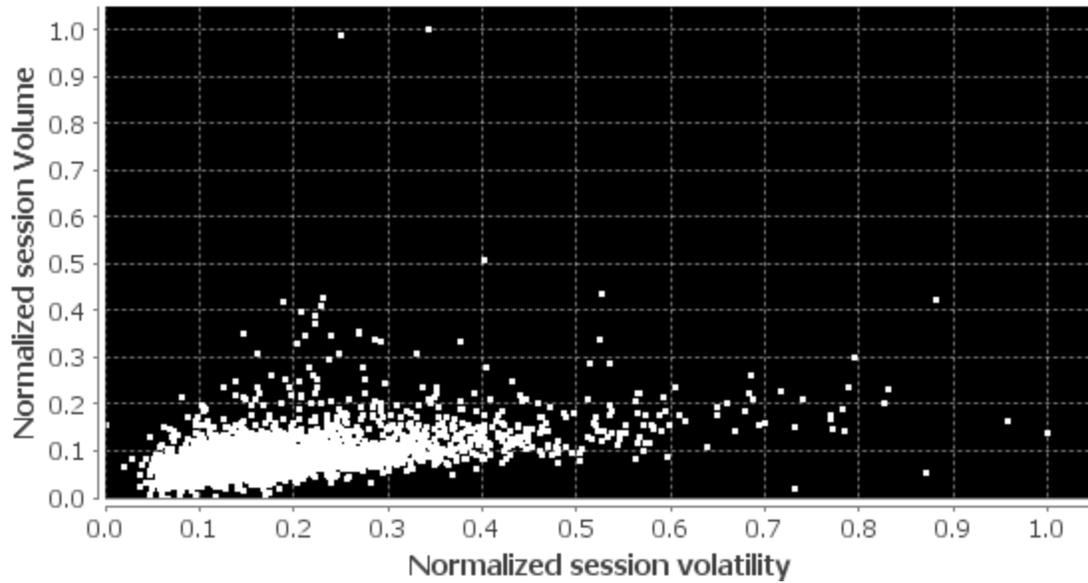
$$w_i^{(t+1)} = w_i^{(t)} + \eta \frac{x_i y}{1 + e^{w^T x}}$$



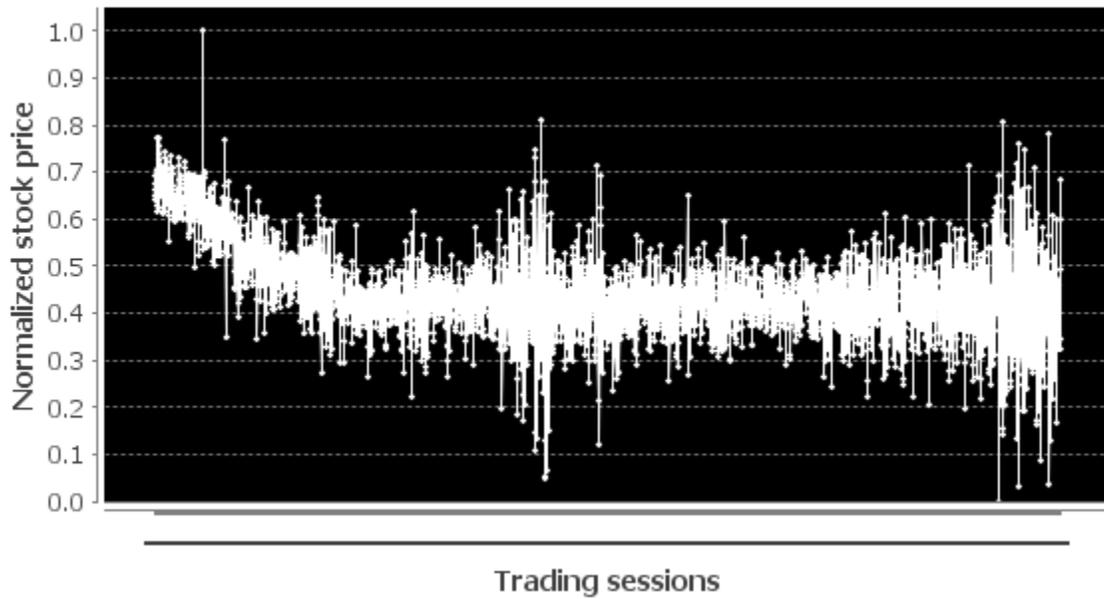


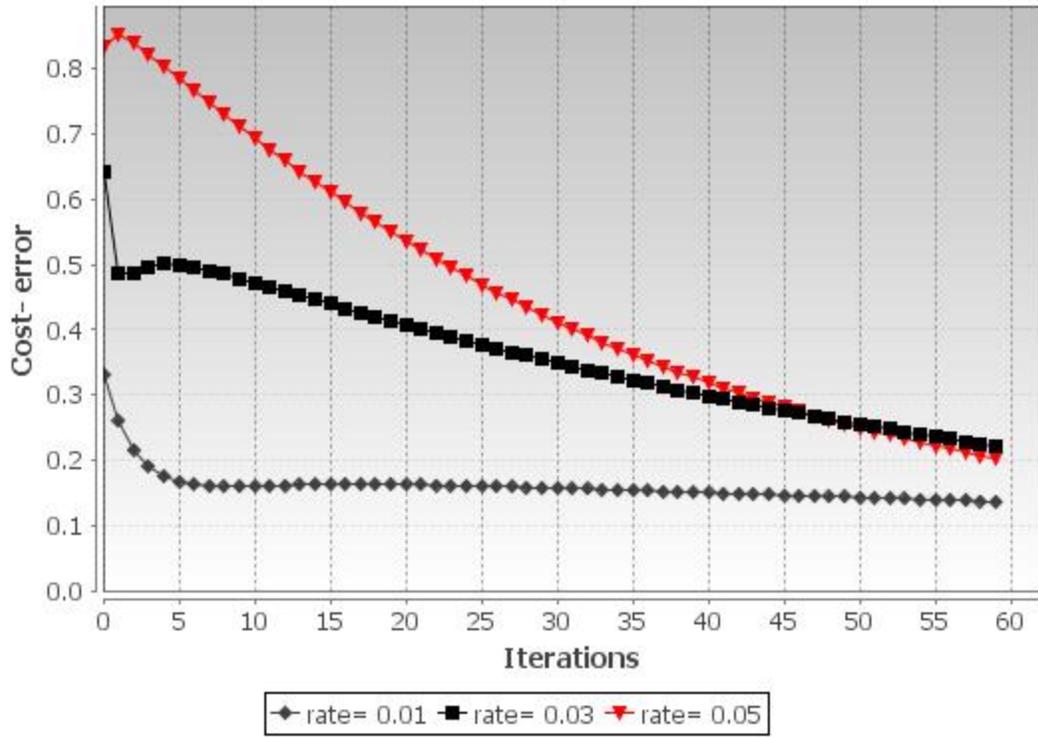


CSCO 2012-13: Model features

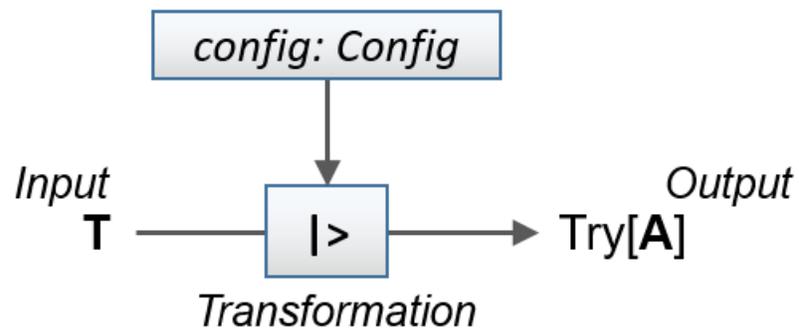
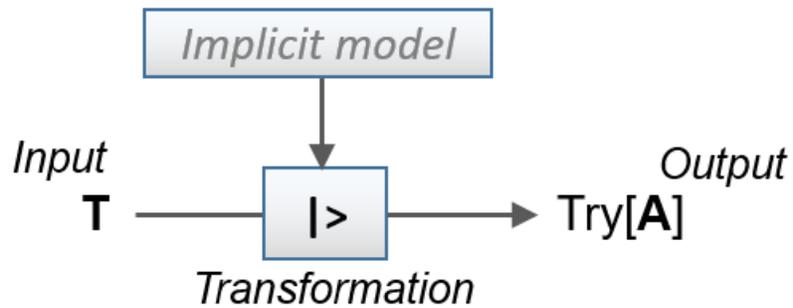
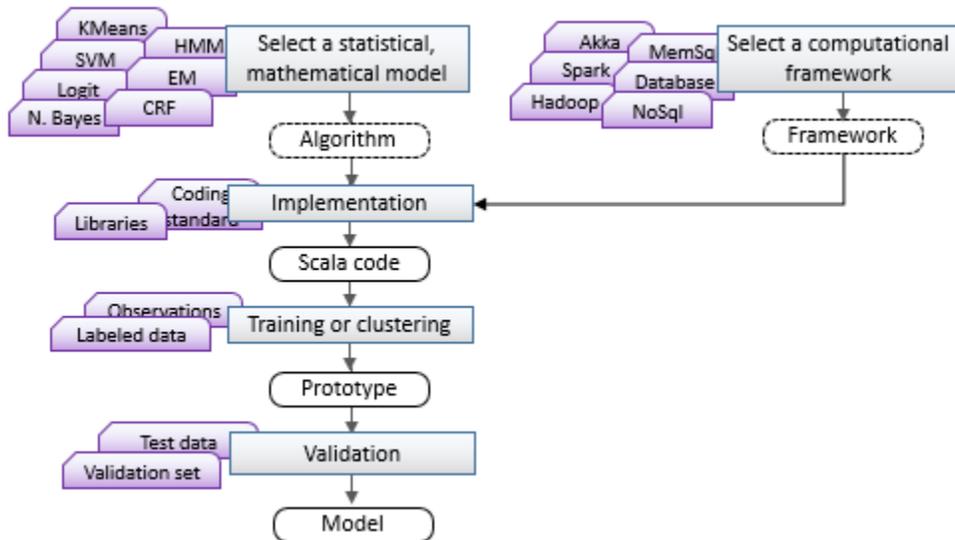


CSCO 2012-13: Training label



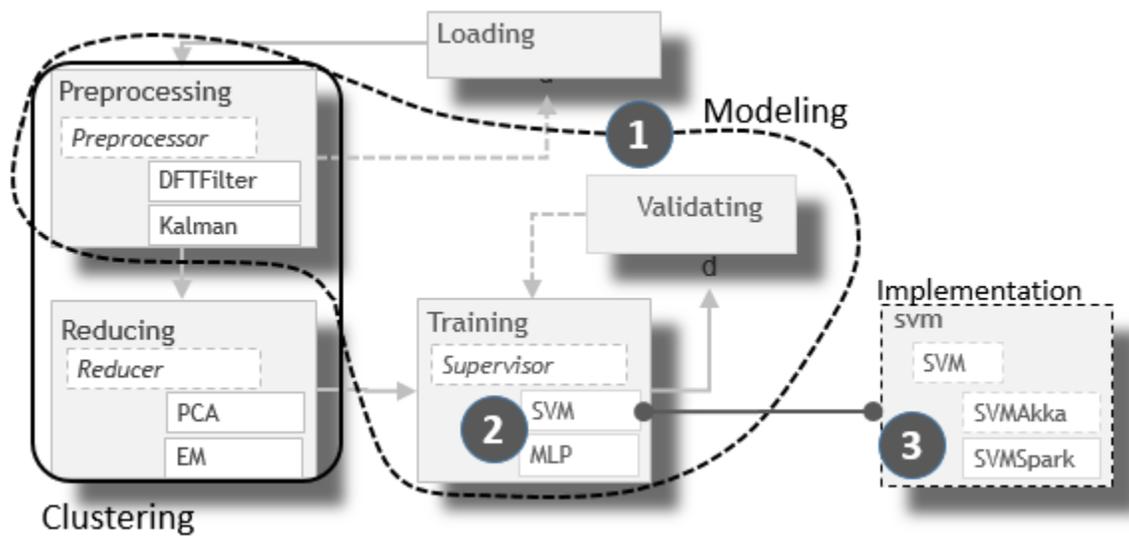
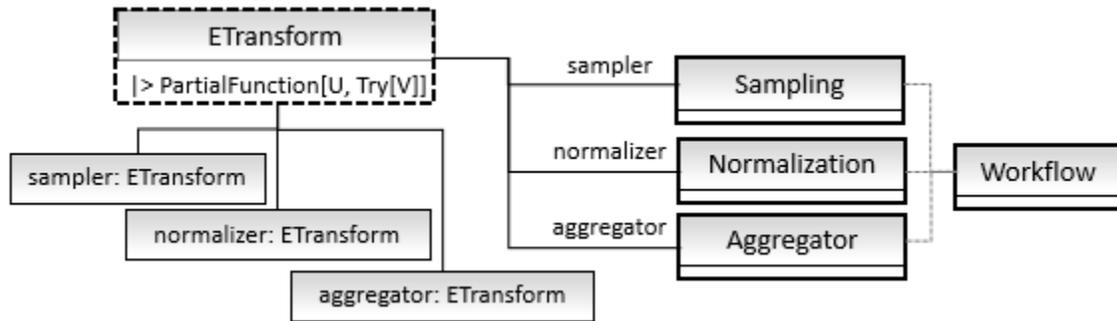


Chapter 2: Data Pipelines



$$f: \mathbb{R}^n \rightarrow \mathbb{R}^n \quad g: \mathbb{R}^n \rightarrow \mathbb{R}$$

$$f: x \rightarrow e^x \quad g: x \rightarrow \sum_0^{n-1} x_i$$



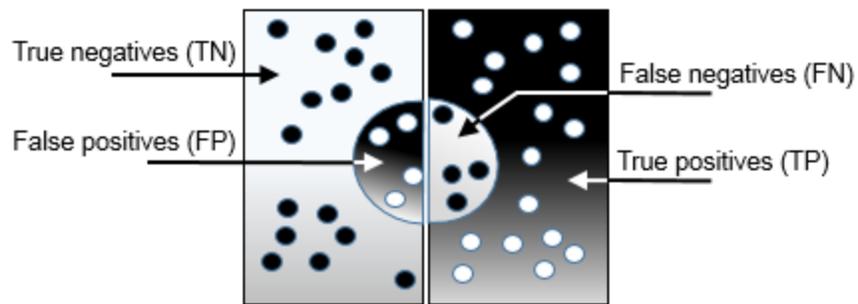
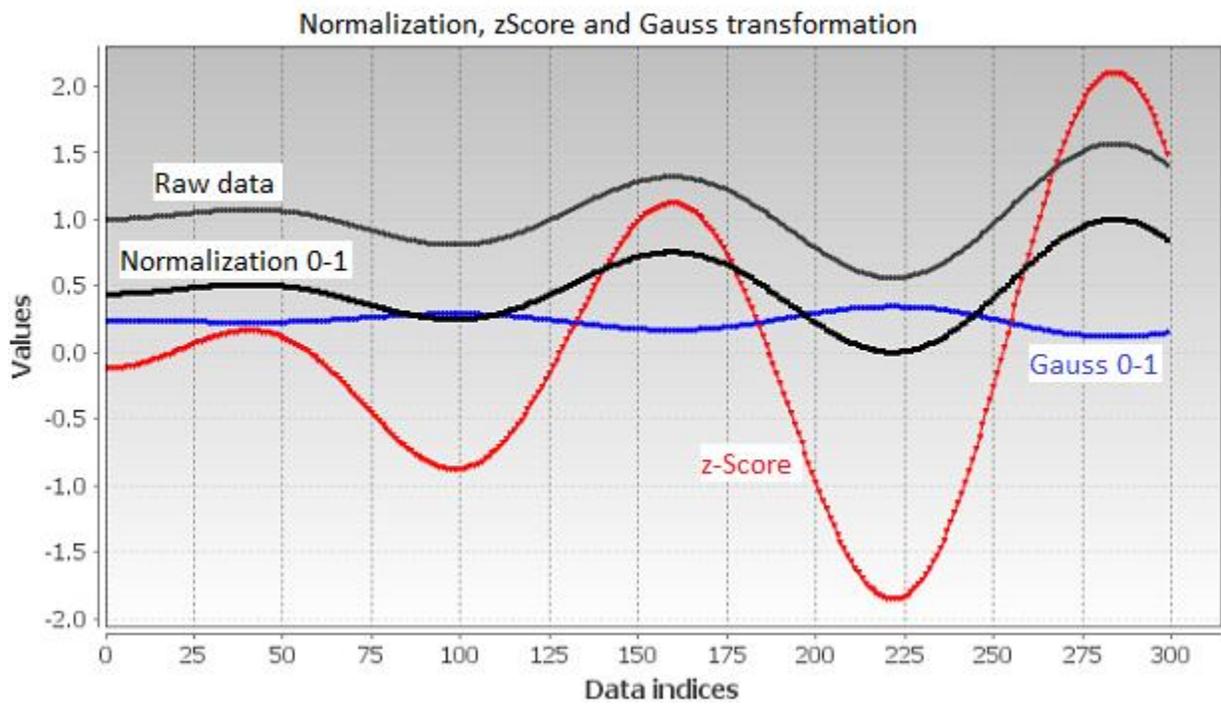
$$E[X] = \mu = \frac{1}{n} \sum_{i=0}^{n-1} x_i$$

$$Var(X) = \frac{\sum (E(X) - x_j)^2}{n-1}$$

$$\overline{Var}(X) = \frac{1}{n-1} \sum_{i=1}^n (x_i - E[X])^2$$

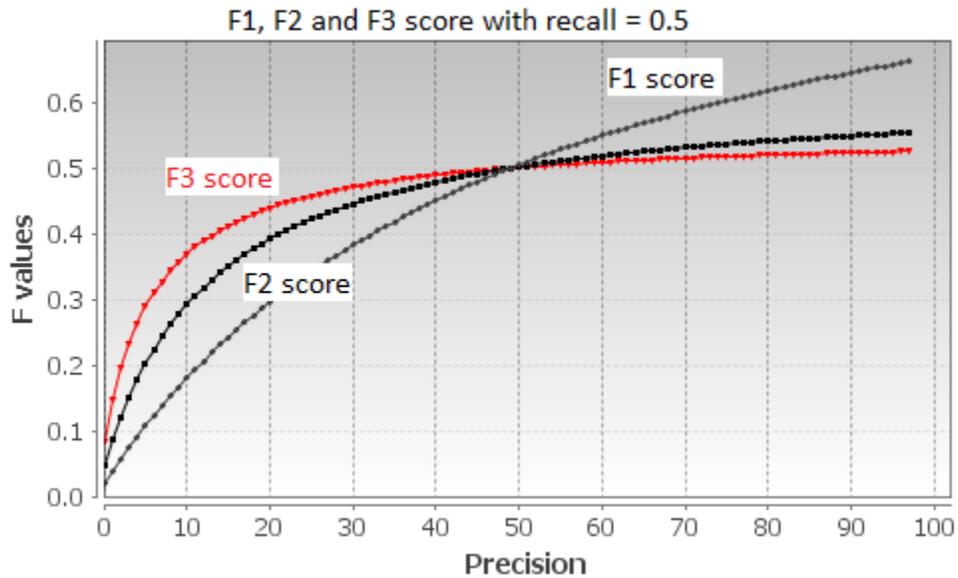
$$y = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$z_i = \frac{x_i - \mu}{\sigma}$$



$$ac = \frac{tp + tn}{tp + tn + fp + fn} \quad p = \frac{tp}{tp + fp} \quad r = \frac{tp}{tp + fn}$$

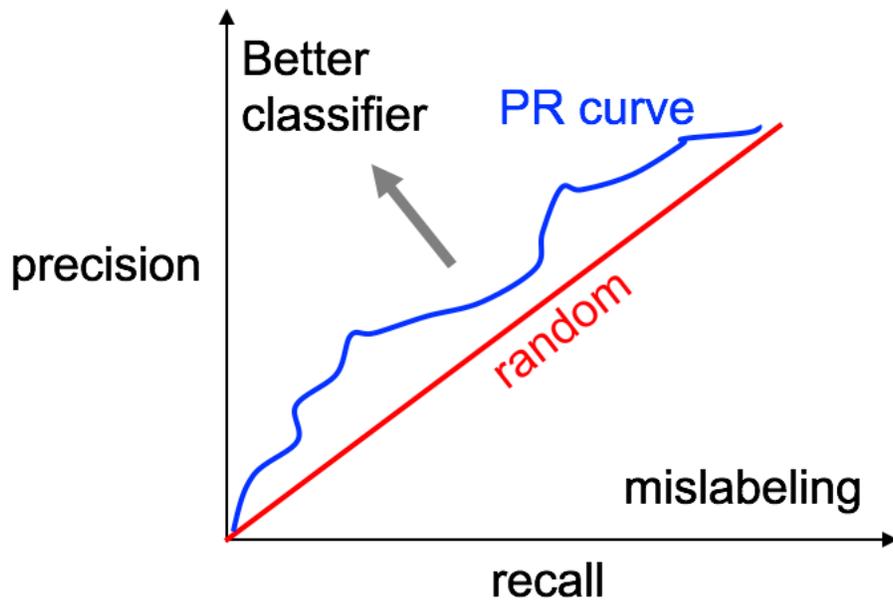
$$F_1 = \frac{2pr}{p+r} \quad F_n = \frac{(1+n^2)pr}{n^2p+r} \quad G = \sqrt{pr}$$



$$p^* = \frac{1}{c} \sum_{i=0}^{c-1} \frac{tp_i}{tp_i + fp_i} \quad r^* = \frac{1}{c} \sum_{i=0}^{c-1} \frac{tp_i}{tp_i + fn_i}$$

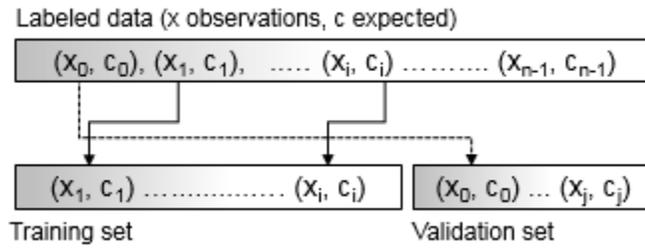
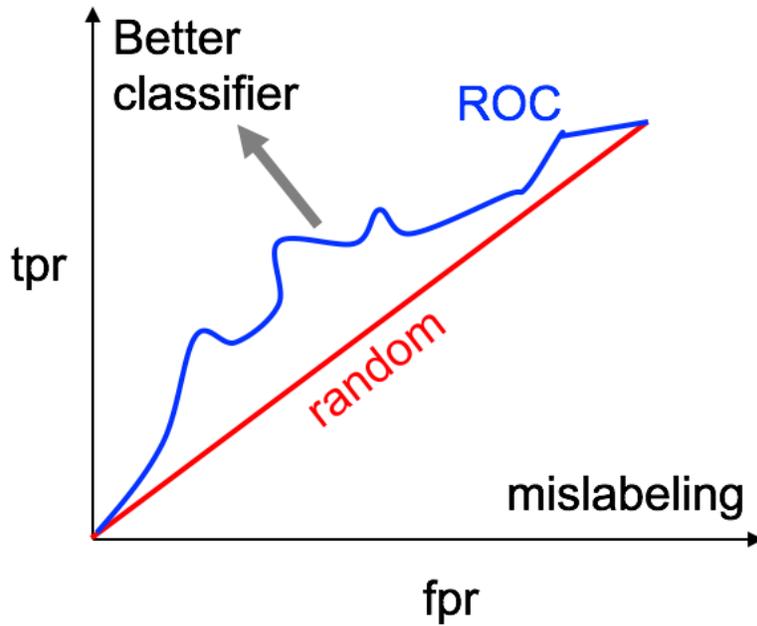
		Actual					
		1	2	3	4	5	6
Predicted	1	167	3	19	8	0	2
	2	11	107	3	27	4	12
	3	4	21	145	3	7	14
	4	9	17	4	179	20	0
	5	15	0	18	2	139	8
	6	1	6	0	24	8	164

Annotations: True positive (diagonal elements), False positives (off-diagonal elements in the top row), False negatives (off-diagonal elements in the first column).



$$auPRC = 0.5 + \frac{1}{N} \sum_{i=0}^{N-1} (p_i - r_i)$$

$$trp = \frac{tp}{tp + fn} \quad fpr = \frac{fp}{fp + tn}$$



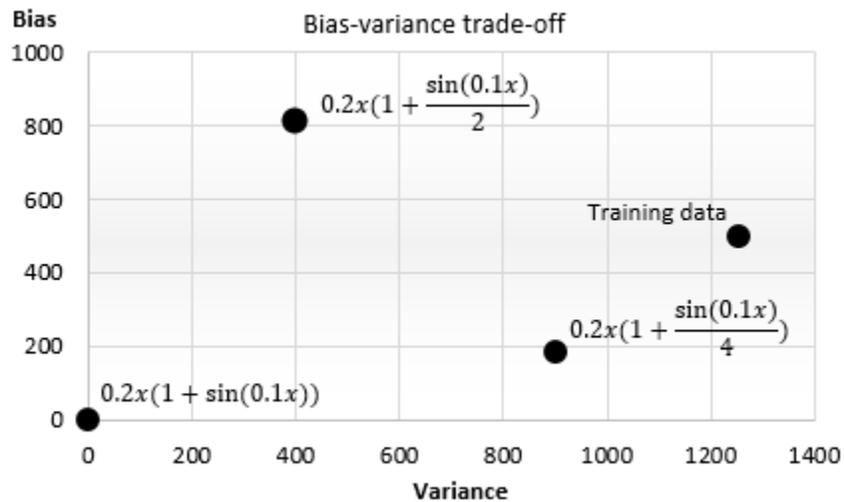
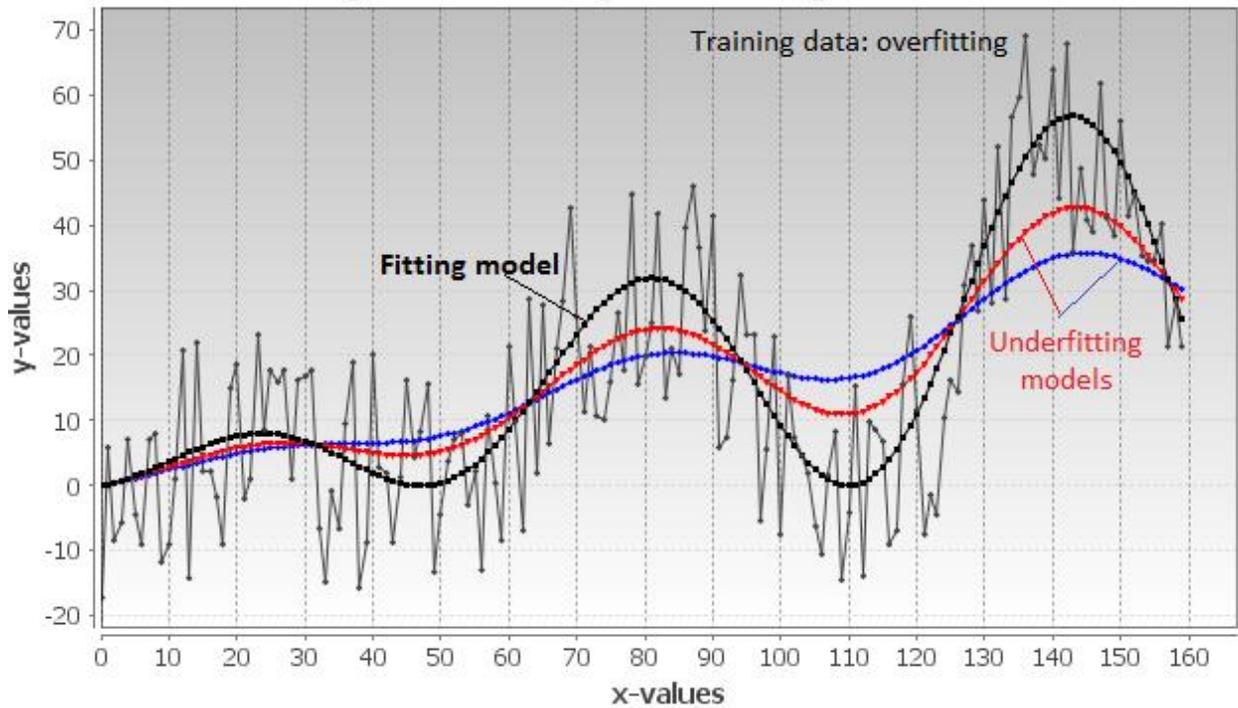
$$var \hat{\theta} = E \left[\left(\hat{\theta} - E \left[\hat{\theta} \right] \right)^2 \right] \quad bias \hat{\theta} = \hat{\theta} - \theta$$

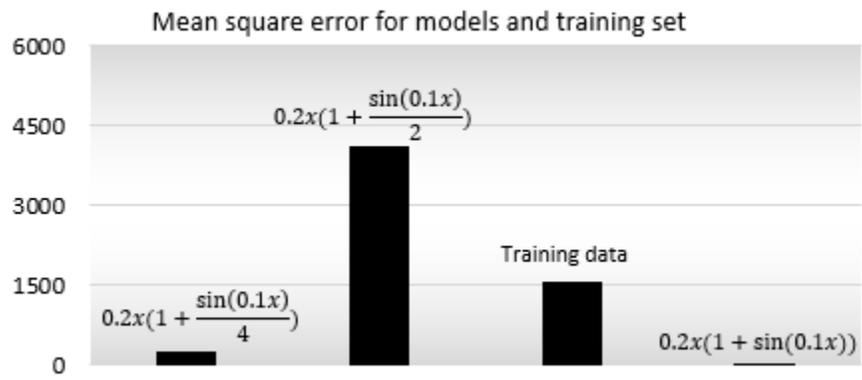
$$MSE = var(\tilde{\theta}) + bias(\tilde{\theta})$$

$$y = \frac{x}{5} \left(1 + \frac{1}{n} \sin\left(\frac{x}{10} + r1\right) \right) + r2$$

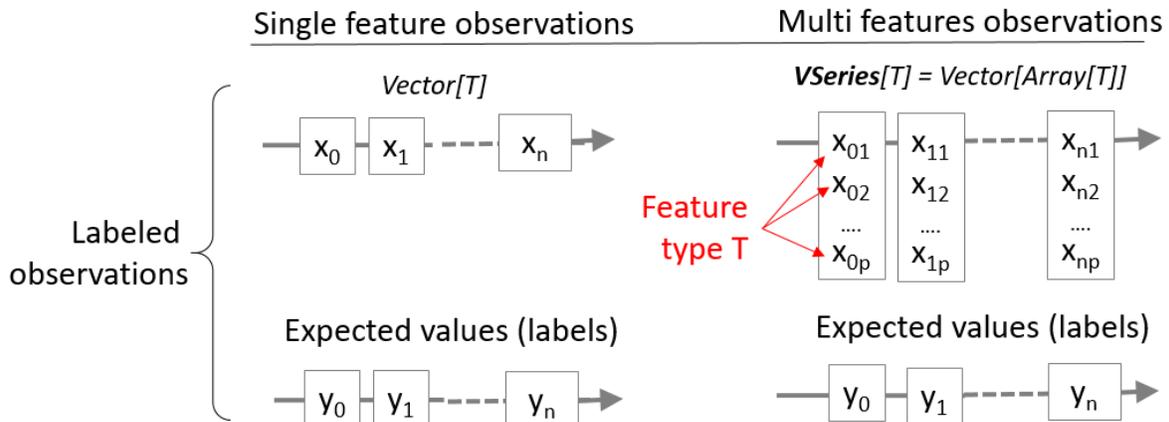
$$y = \frac{x}{5} \left(1 + \frac{1}{n} \sin\left(\frac{x}{10}\right) \right)$$

Training set with overfitting and underfitting models





Chapter 3: Data Pre-processing

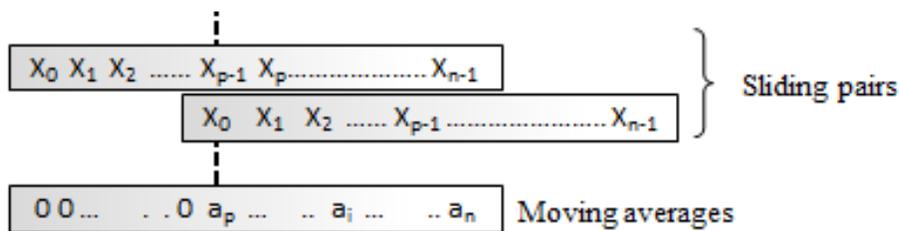


$$\tilde{x}_t = f(x_{t-p+1}, \dots, x_t) \quad \forall t \geq p$$

$$\tilde{x}_t = \frac{1}{p} \sum_{j=t-p+1}^t x_j \quad \forall t \geq p$$

$$0 \quad \forall t < p$$

$$\tilde{x}_t = \tilde{x}_{t-1} + \frac{1}{p} (x_t - x_{t-p}) \quad \forall t \geq p$$

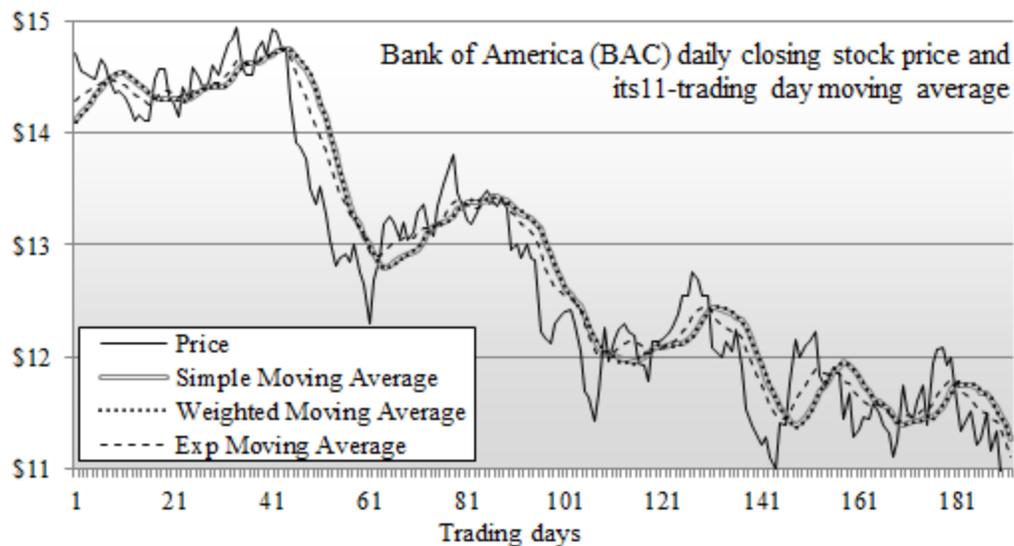
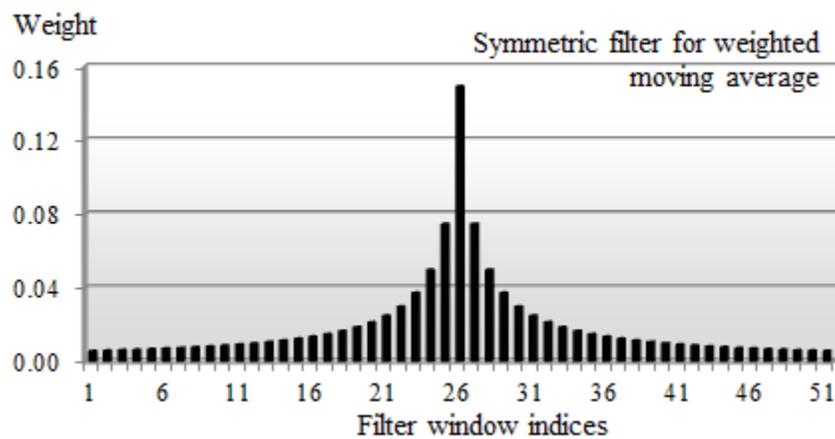


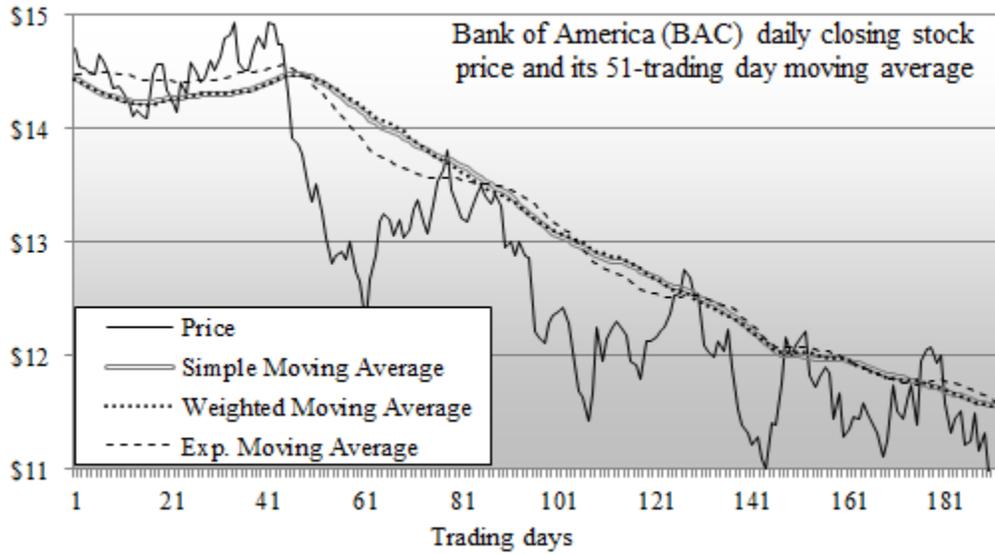
$$\tilde{x}_t = \sum_{j=t-p+1}^t \alpha_{j-t+p} x_j \quad \forall t \geq p \quad \text{subject to} \quad \sum_{i=0}^{p-1} \alpha_i = 1$$

$$0 \quad \forall t < p$$

$$\tilde{x}_t = (1 - \alpha) \tilde{x}_{t-1} + \alpha x_t \quad \forall t > 0 \quad 0 < \alpha < 1$$

$$\tilde{x}_0 = x_0$$





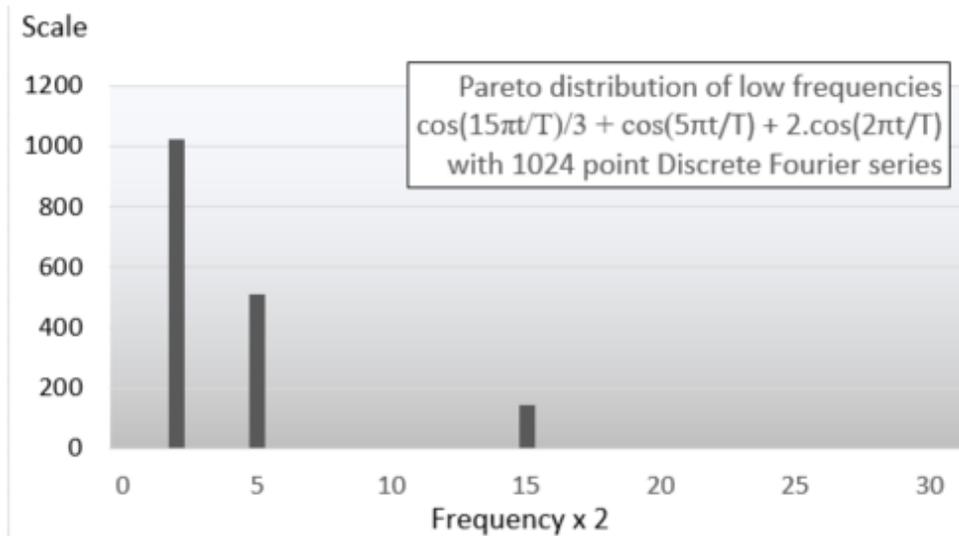
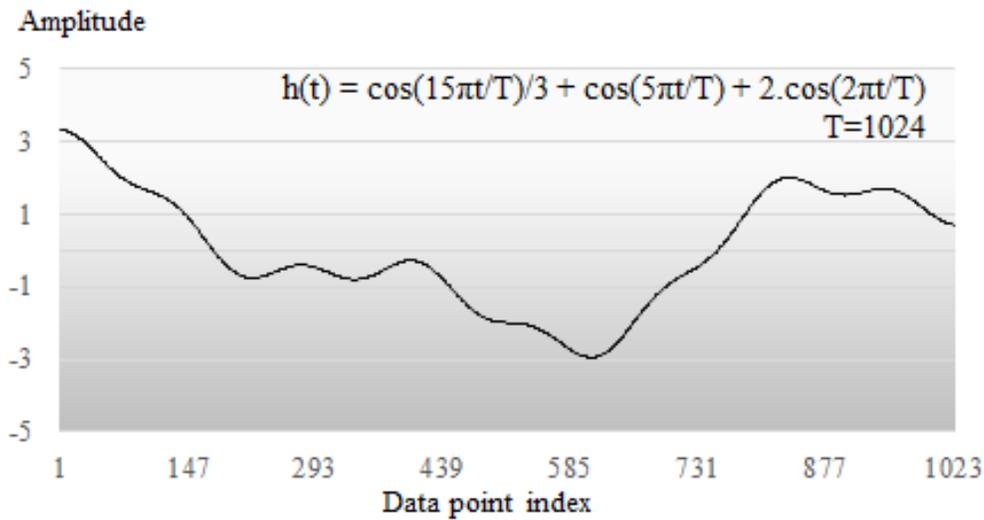
$$f(t) = \frac{a_0}{2} + \sum_1^{\infty} a_k \cos(nx) + \sum_1^{\infty} b_k \sin(nx)$$

$$\mathcal{F}^c(f, k) = \int_{-\infty}^{\infty} \cos(2\pi kx) f(x) dx$$

$$f(x) = f(-x) = \frac{a_0}{2} + \sum_{k=1}^{2N-3} a_k \cos(kx) \quad \text{where } a_k = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(kt). dt$$

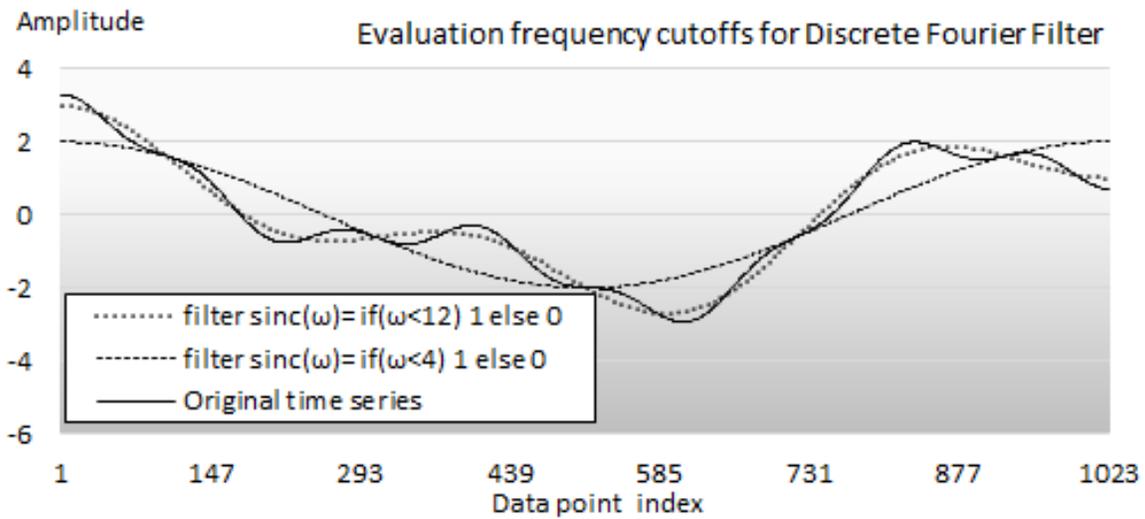
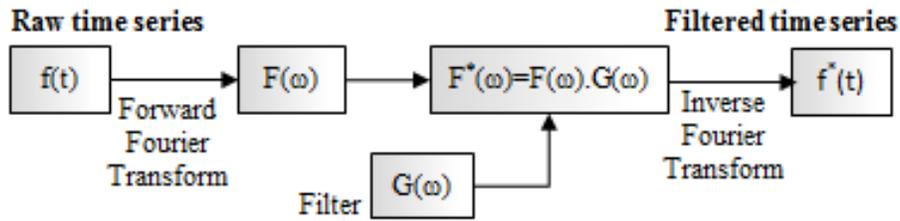
$$\mathcal{F}^s(f, k) = \int_{-\infty}^{\infty} \sin(2\pi kx) f(x) dx$$

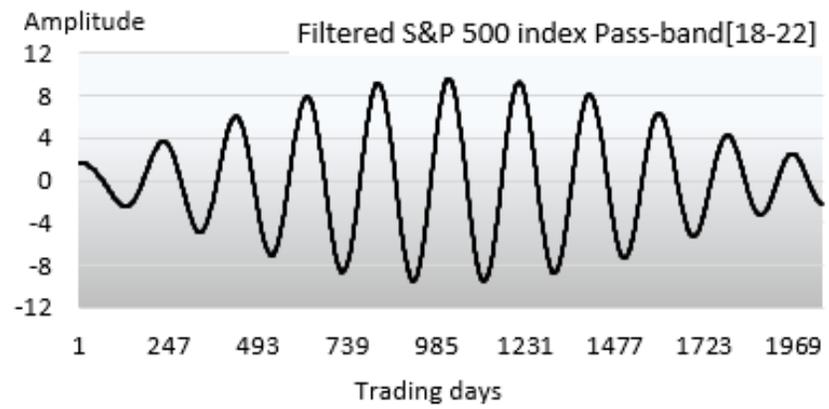
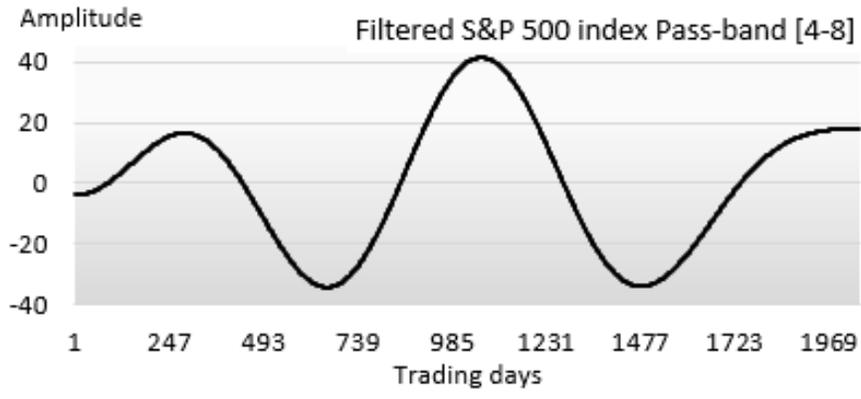
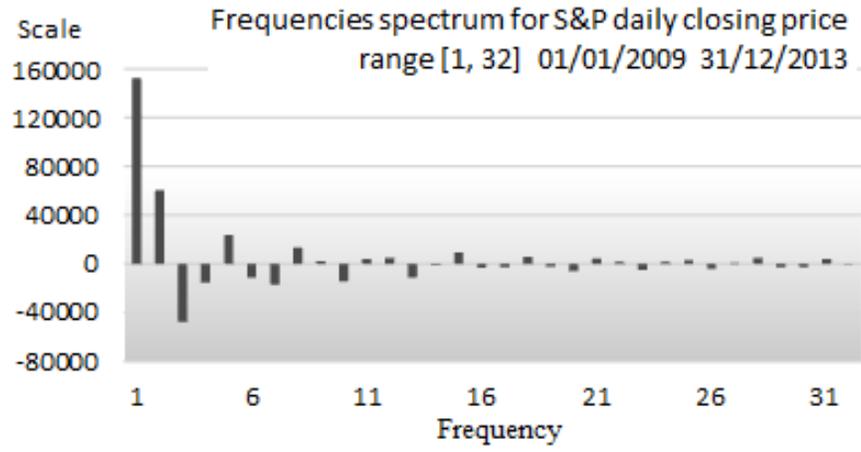
$$f(x) = -f(-x) = \sum_{k=1}^{2N-3} b_k \sin(kx) \quad \text{where } b_k = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(kt) . dt$$

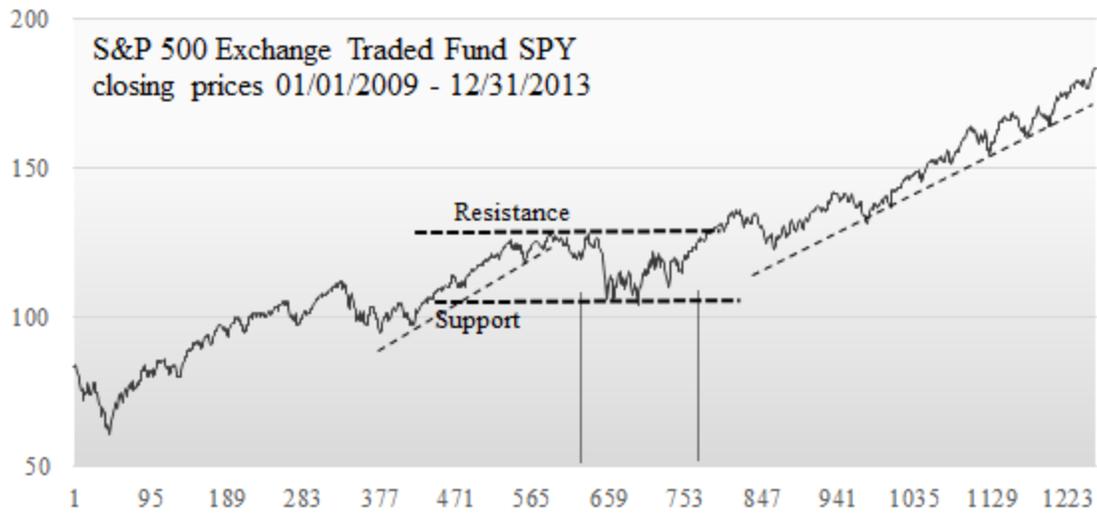


$$\langle f, g \rangle = \int_{-\infty}^{\pi} f(t) . g(x-t) dt$$

$$F(x * f) = F(x) \cdot F(f) = \sum_{j=0}^{N-1} \omega_j^x \cdot \omega_{k-j}^f$$

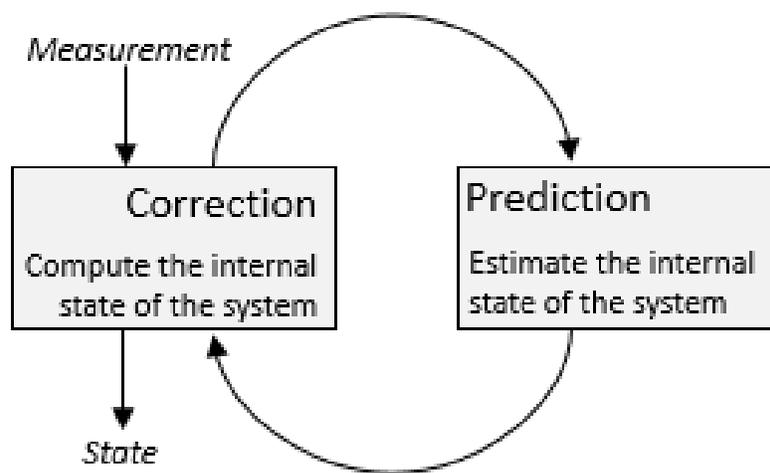






$$x_t = A_t \cdot x_{t-1} + B_t \cdot u_t + w_t$$

$$z_t = H_t \cdot x_t + v_t$$



$$\begin{bmatrix} \hat{x}_t \\ \hat{x}_{t-1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_t \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} w_t \\ w_{t-1} \end{bmatrix}$$

$$\hat{x}_t' = A_t \cdot \hat{x}_{t-1} + B_t \cdot u_t$$

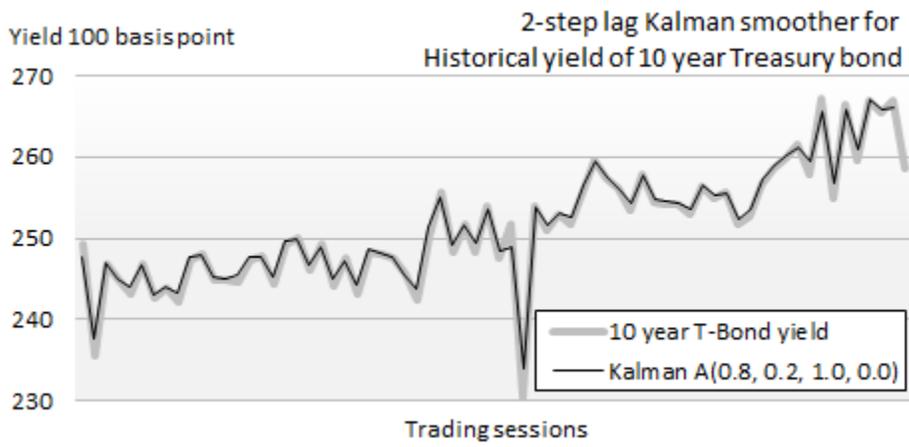
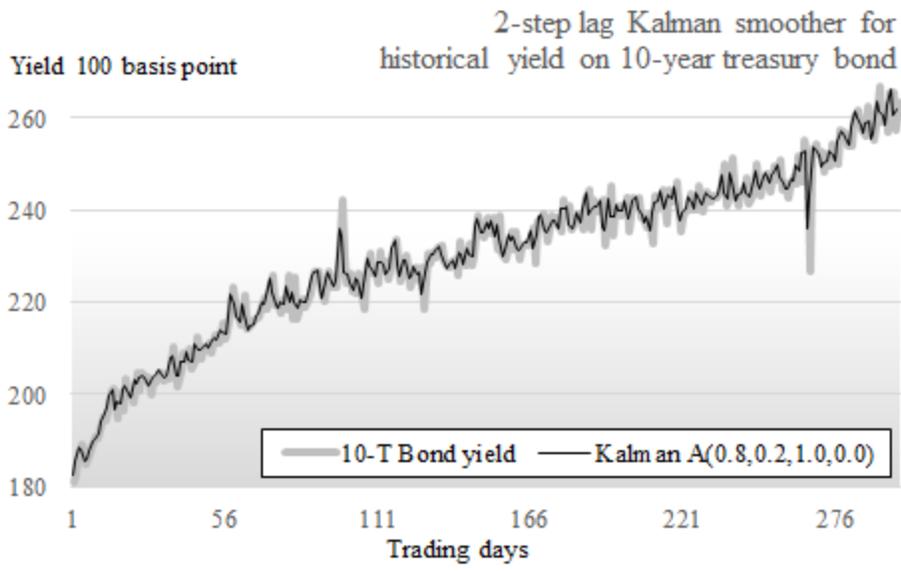
$$P_t' = A_t \cdot P_{t-1} + A_t^T + Q_t$$

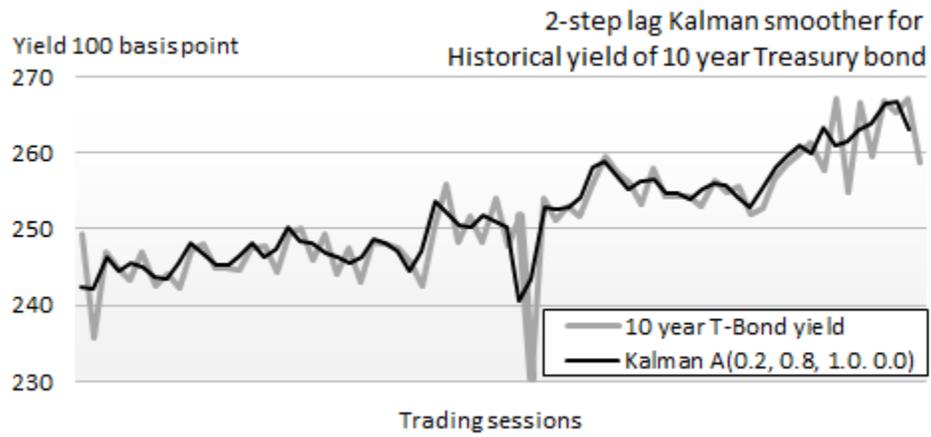
$$\begin{bmatrix} z_t \\ z_{t-1} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_t \\ \hat{x}_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ v_{t-1} \end{bmatrix}$$

$$\hat{x}_t = \hat{x}_t' + K_t (z_t - H_t \cdot \hat{x}_t') \quad r_t = z_t - H_t \cdot \hat{x}_t'$$

$$K_t = P_t' \cdot H_t^T (H_t \cdot P_t' \cdot H_t^T + R_t)^{-1}$$

$$S_t = [x_{t+1}, x_t]^T \text{ with } \begin{vmatrix} x_{t+1} \\ x_t \end{vmatrix} = \begin{vmatrix} \alpha & 1-\alpha \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x_t \\ x_{t-1} \end{vmatrix}$$





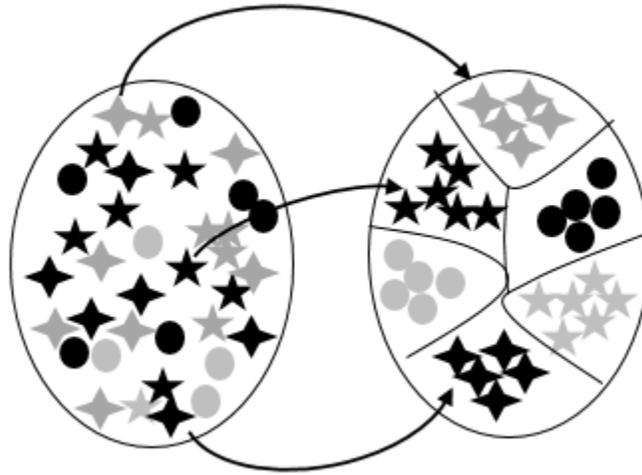
$$\{x_{t+1} - x_t\}$$

$$\{x_t\}$$

$$\tilde{x}_t$$

$$\tilde{x}$$

Chapter 4: Unsupervised Learning

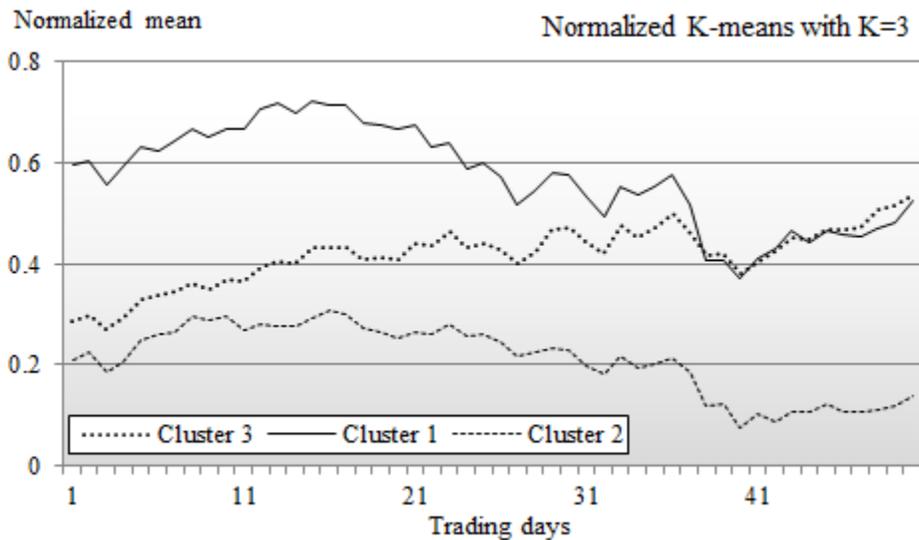
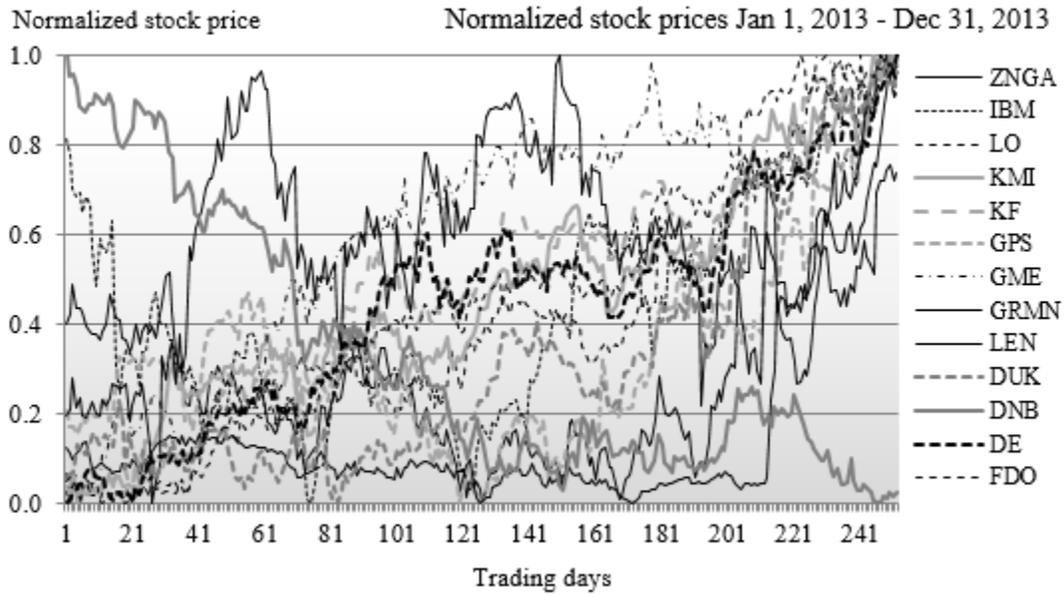


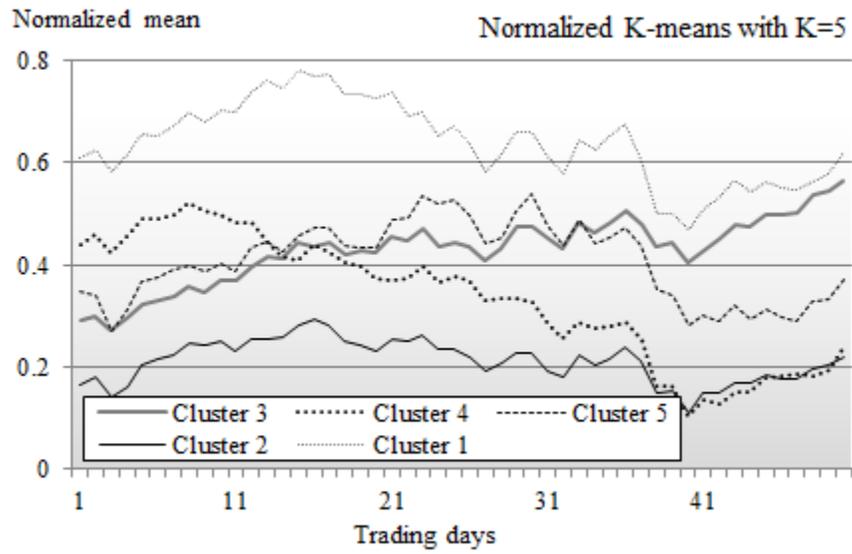
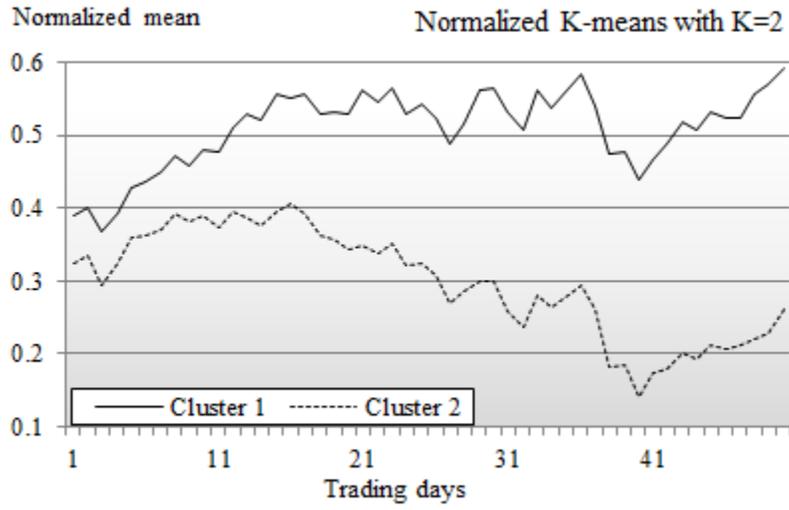
$$d(x, y) = \sum |x_i - y_i|$$

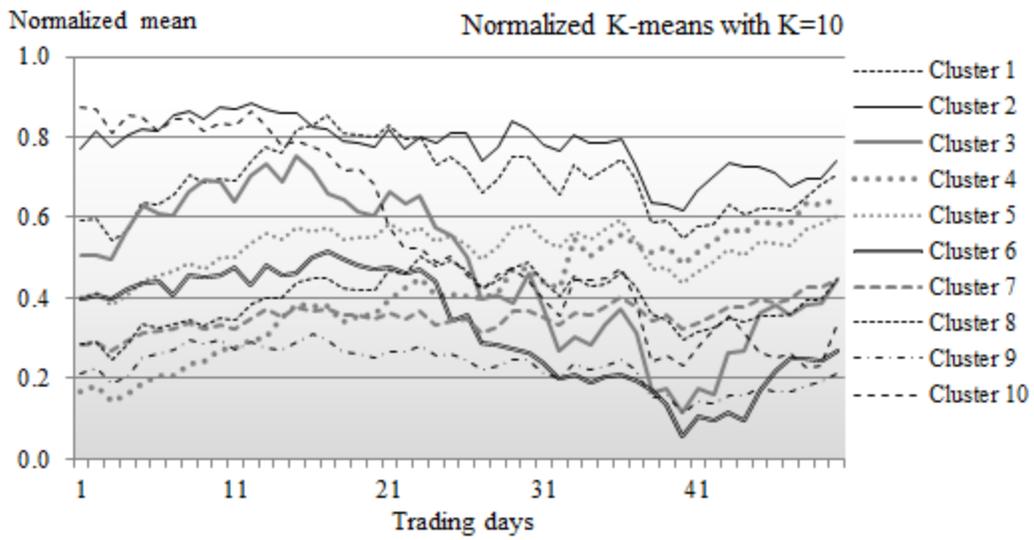
$$d(x, y) = \sum (x_i - y_i)^2$$

$$d(x, y) = \frac{\sum x_i y_i}{(\sum x_i^2 \sum y_i^2)^{1/2}}$$

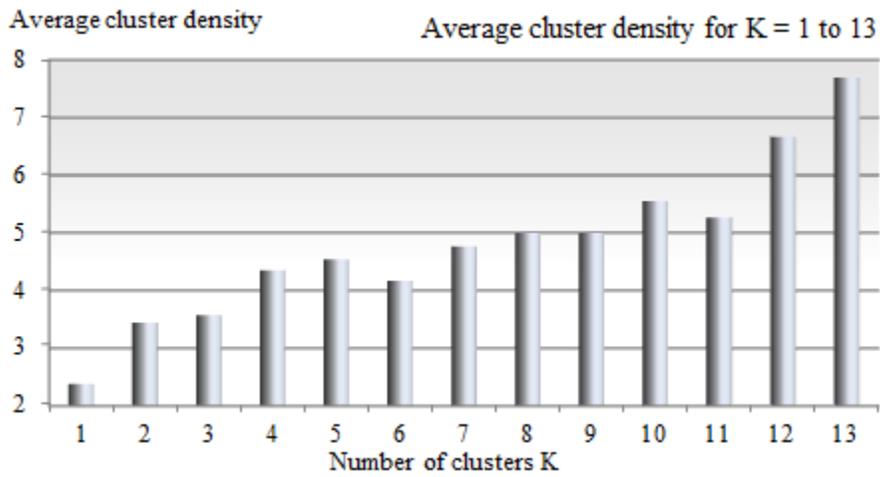
$$\min_{C_k} \sum_1^K \sum_{x_i \in C_k} d(x_i, m_k)$$



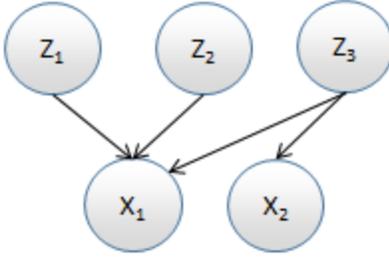




$$d(C_j) = 1 / \sum_{x \in C_j} (x - c_j)^2$$



$$H(p) = - \sum_c p_c \cdot \log_2 p_c$$



$$p(x_i | \theta) = \sum_z p(x_i, z | \theta)$$

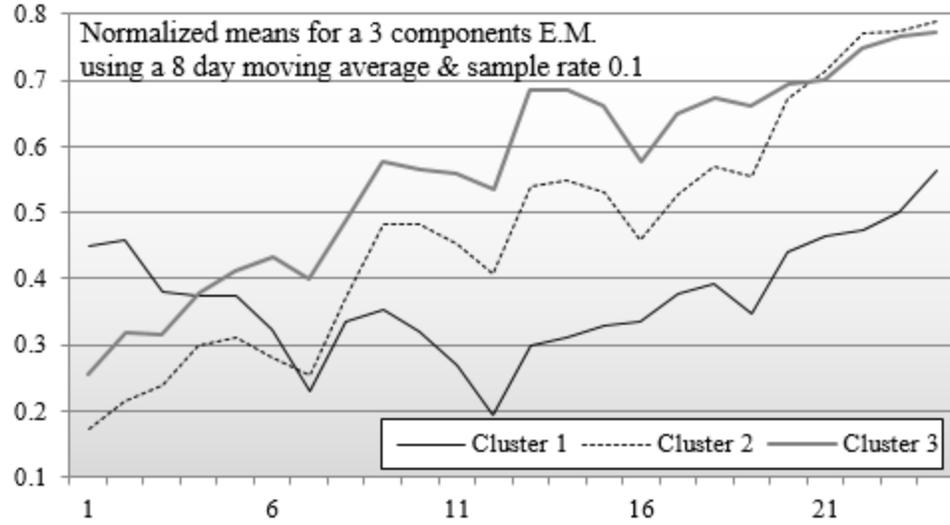
$$\mathcal{L}(\theta) = \sum_{i=0}^{N-1} \log \left\{ \sum_z p(x_i, z | \theta) \right\} \quad \tilde{\theta} = \operatorname{argmax} \mathcal{L}(\theta)$$

$$\mathbb{Q}(\theta, \theta^n) = \sum_z p(z | x_i, \theta^n) \cdot \log p(x_i, z | \theta)$$

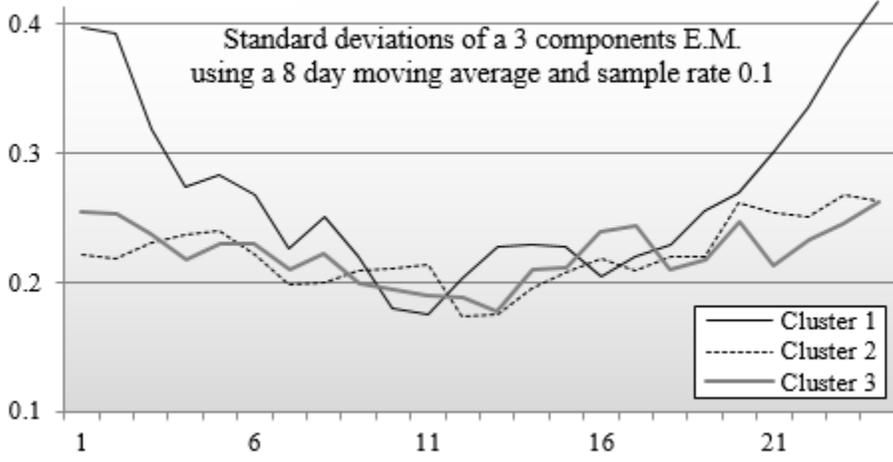
Q

$$\theta^{n+1} = \operatorname{arg max}_{\theta} \mathbb{Q}(\theta, \theta^n) \quad |\theta^{n+1} - \theta^n| < \varepsilon$$

Normalized mean



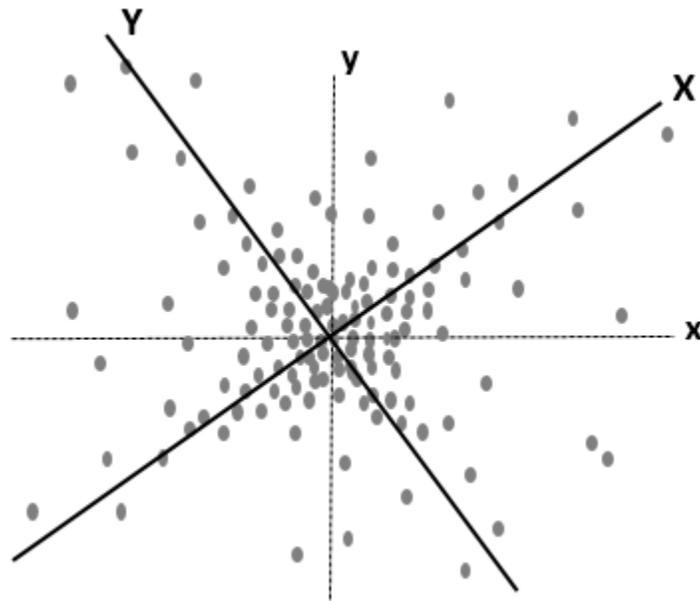
Standard deviation



Chapter 5: Dimension Reduction

$$D_{KL}(P\|Q) = \sum_{i=0}^{n-1} p(x_i) \cdot \log \frac{p(x_i)}{q(x_i)}$$

$$I(X;Y) = D_{kl}[p(X,Y), p(X)p(Y)] = \sum_{i=0}^{n-1} p(x_i, y_i) \log \frac{p(x_i, y_i)}{p(x_i) \cdot p(y_i)}$$



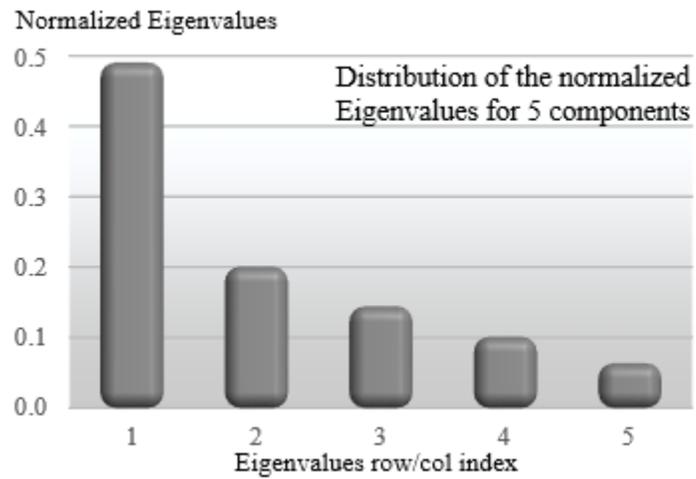
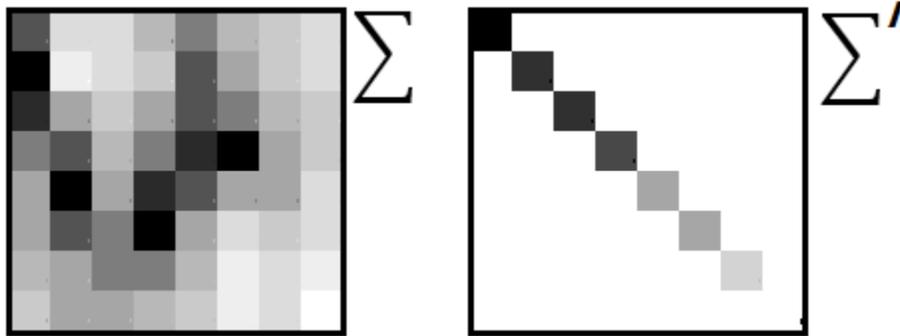
$$M = \left\{ f_{i:1,m} \left| \sum_{i=1}^m \sigma^2(f_i) < \mu \right. \right\}$$

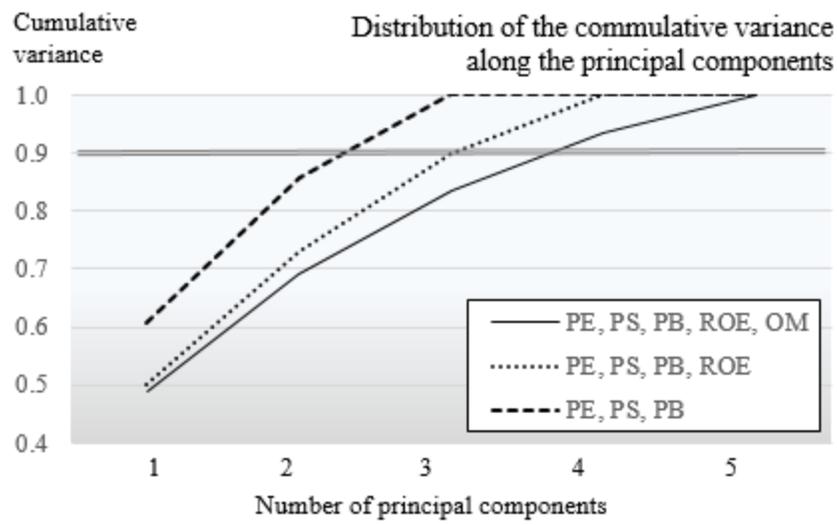
$$\text{cov}(X, Y) = \frac{1}{n-1} \sum_{i=0}^{n-1} (x_i - \bar{x})(y_i - \bar{y})$$

$$x_i \leftarrow \frac{x_i - \bar{x}}{\sigma}$$

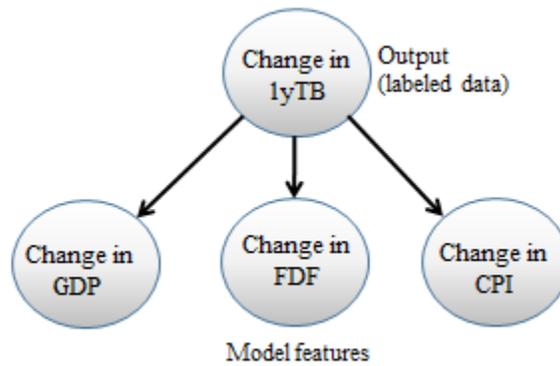
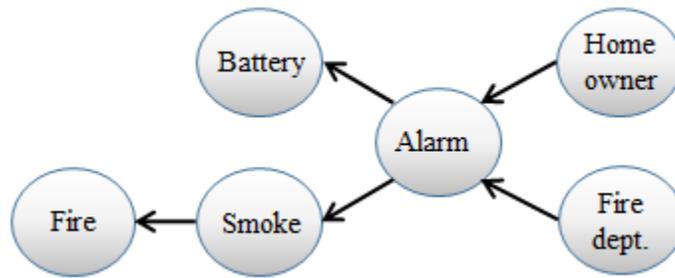
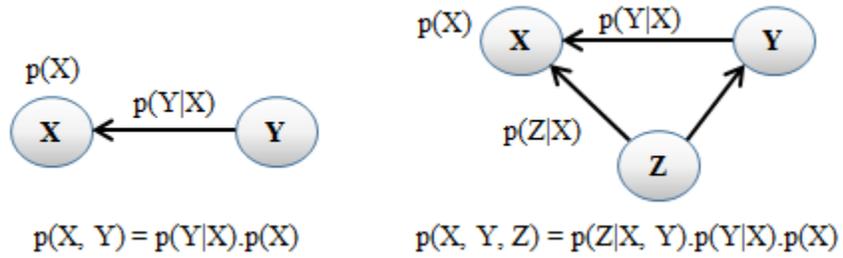
$$\Sigma_{cov} = \begin{vmatrix} cov(x_0, x_0) & \dots & cov(x_0, x_{n-1}) \\ \dots & var(x_i) & \dots \\ cov(x_{n-1}, x_0) & \dots & cov(x_{n-1}, x_{n-1}) \end{vmatrix}$$

$$\Sigma' = W^T \cdot \Sigma_{cov} \cdot W$$





Chapter 6: Naïve Bayes Classifiers

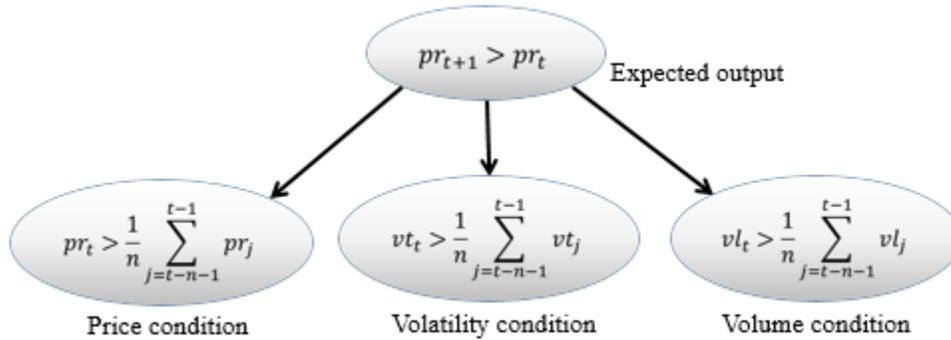
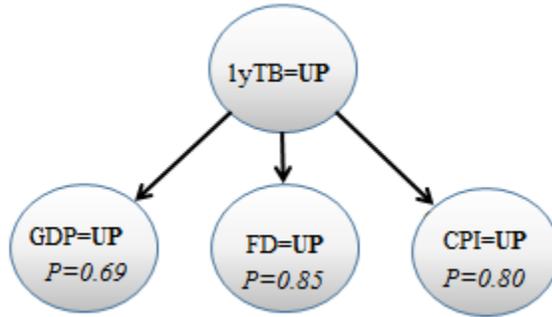


$$p(C_j | x) = \frac{p(x | C_j) \cdot p(C_j)}{p(x)}$$

$$p(C_j | x) = \prod_{i=0}^{n-1} p(x_i | C_j) \cdot p(C_j)$$

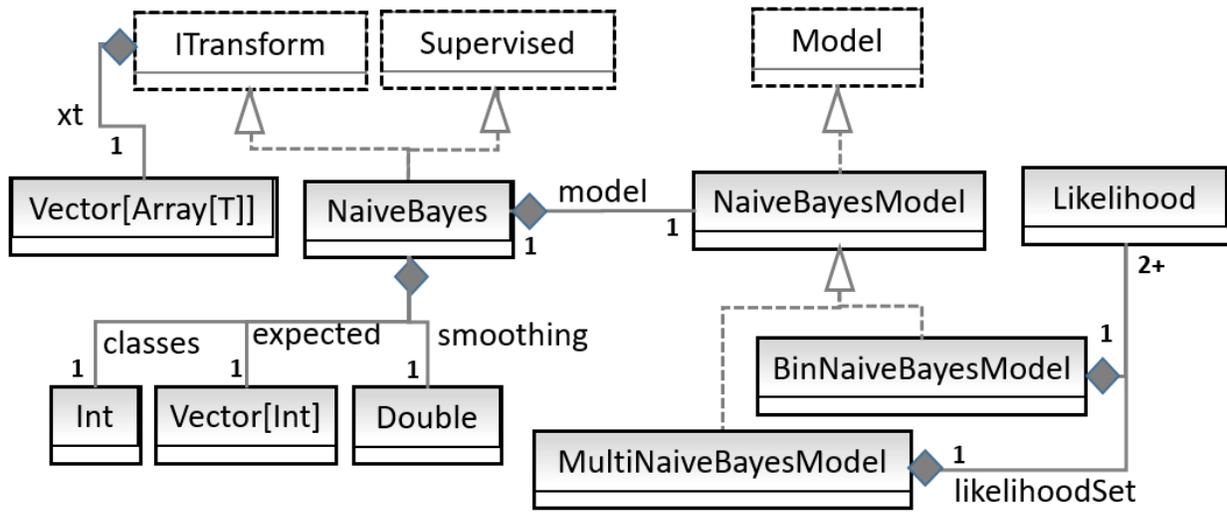
$$\mathcal{L}(C_j | x) = \sum_{i=0}^{n-1} \{ \log p(x_i | C_j) + \log p(C_j) \}$$

$$C_m = \arg \max_j \mathcal{L}(C_j | x)$$

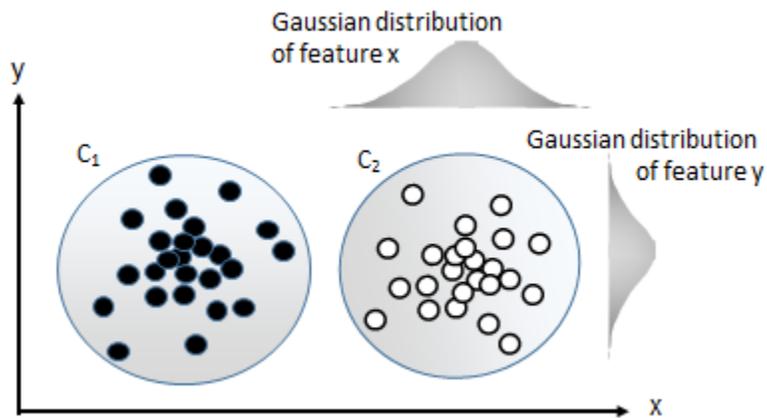


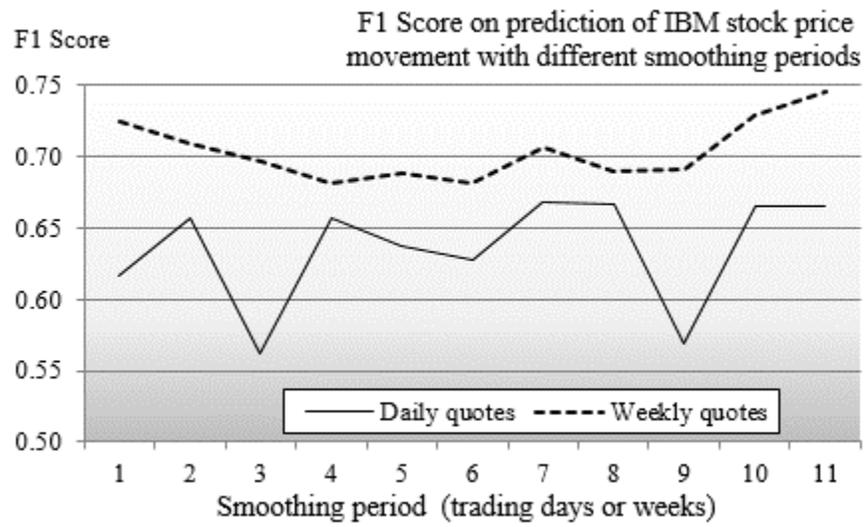
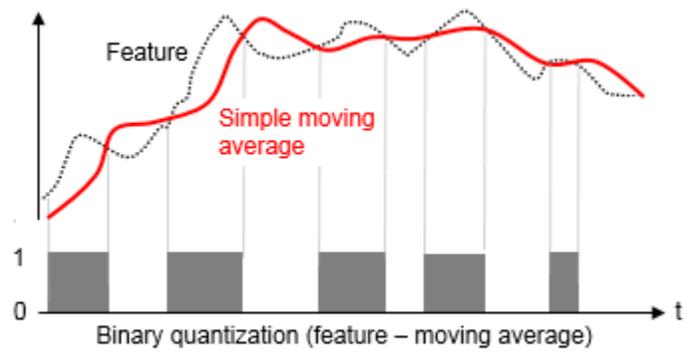
$$\mu' = \frac{k+1}{N+n}$$

$$\mu' = \frac{k+a}{N+\alpha n}$$

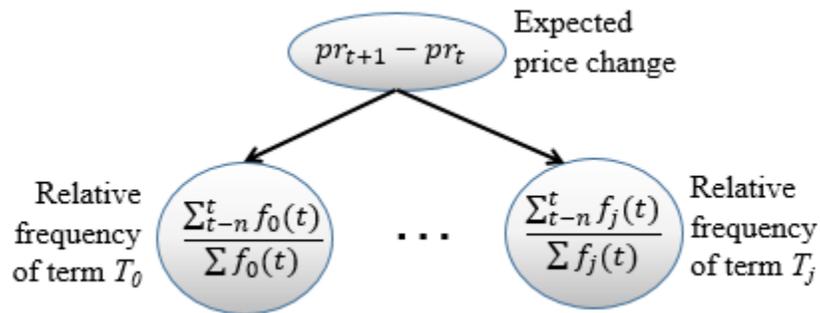
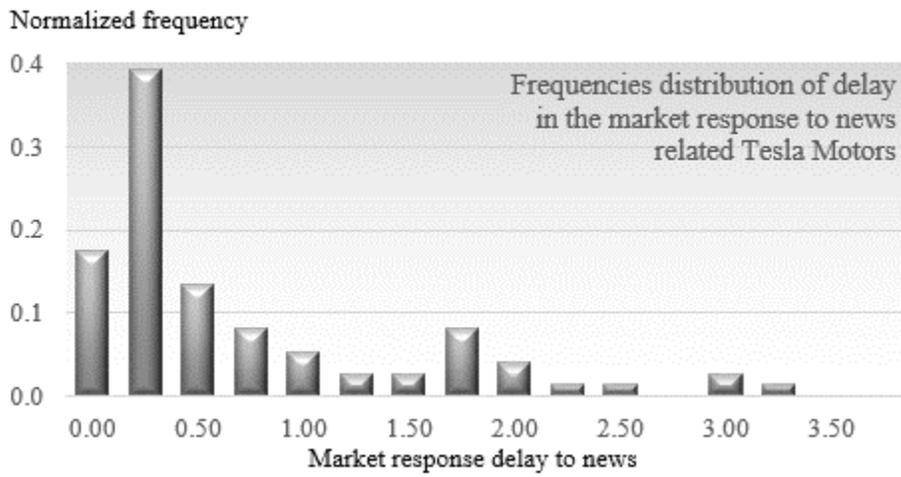
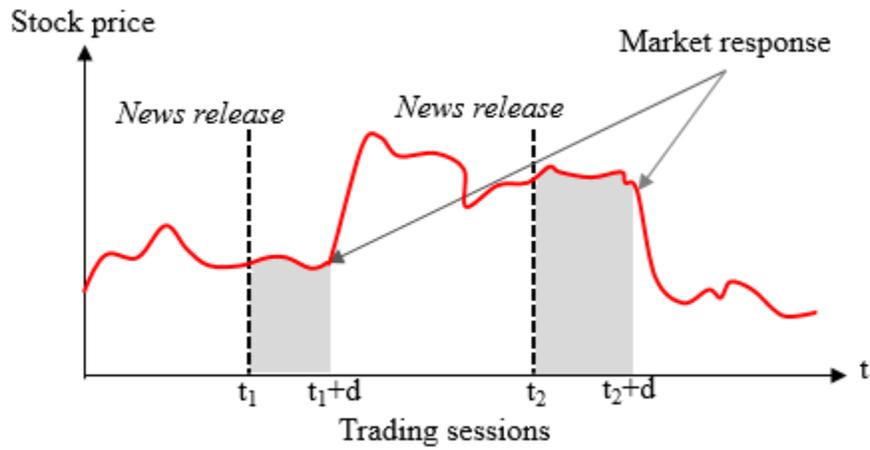


$$\mathcal{L}(C_j | x) = \sum_{i=0}^{n-1} \left[-\frac{1}{2} \log(2\pi) - \log \sigma - \frac{(x_i - \mu')^2}{2\sigma^2} + \log p(C_j) \right]$$



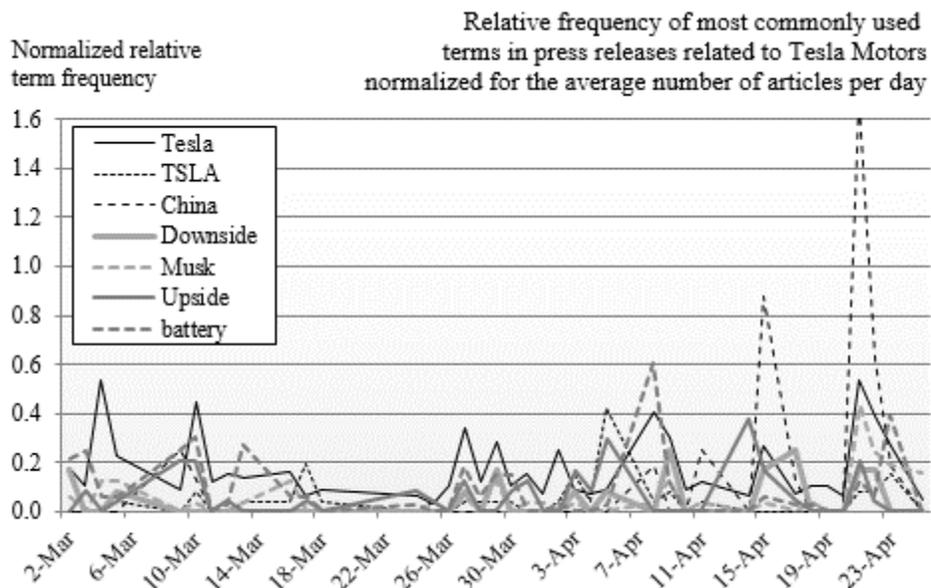
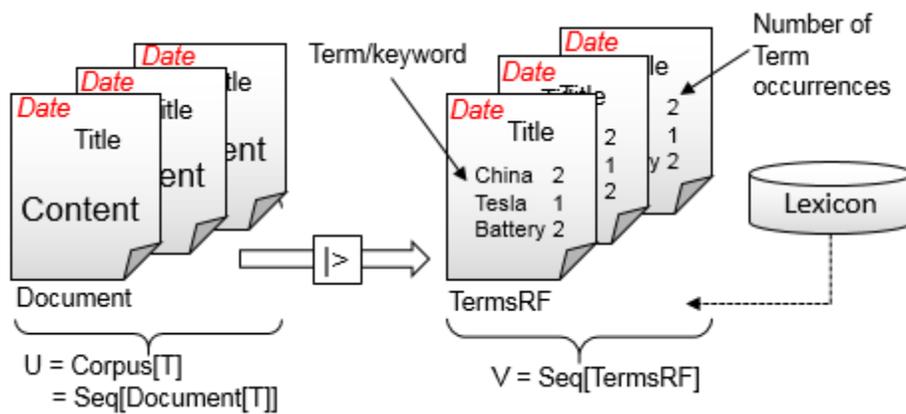


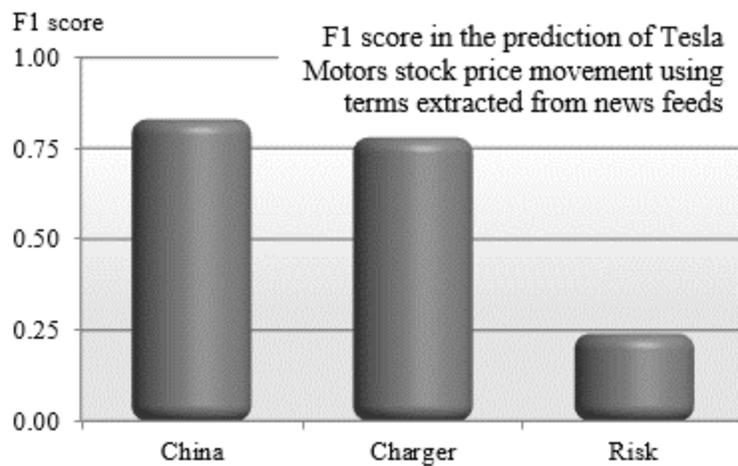
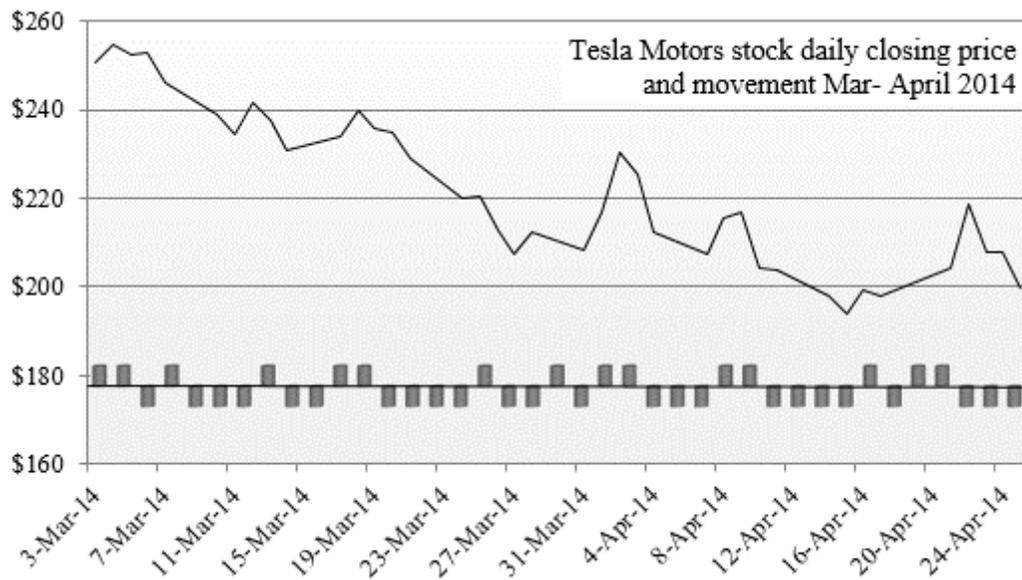
$$p(x|f, C_j) = \prod_{k=0}^{n-1} \{f_k p(x_k | C_j) + (1-f_k)(1-p(x_k | C_j))\}$$



$$rtf \{t_i\} = \frac{\sum_{a \in D_i} n_i^a}{\sum_{a \in Corpus} n_i^a}$$

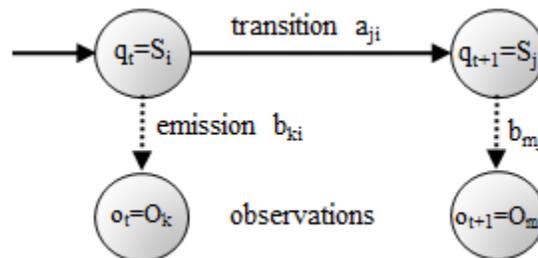
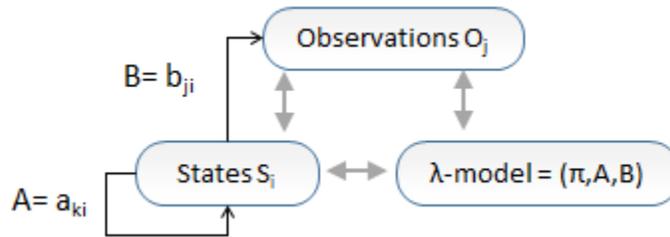
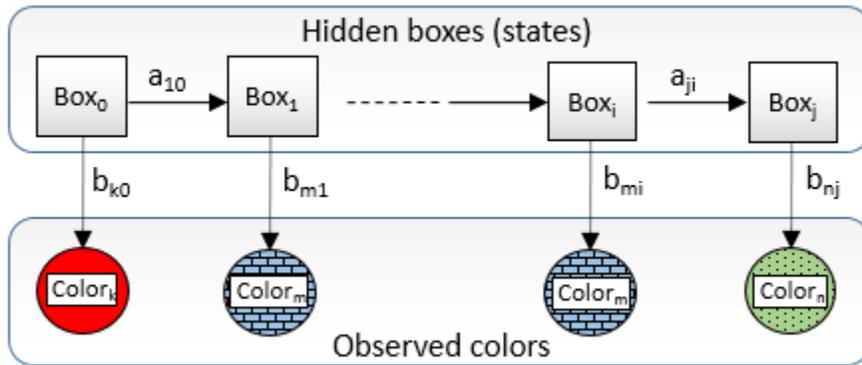
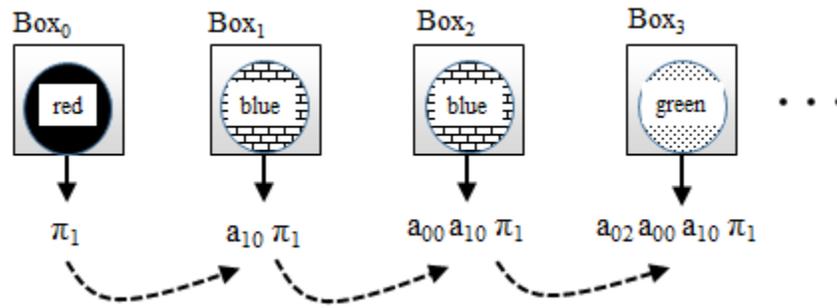
$$nrt \{t_i\} = \frac{rtf \{t_i\} N_d}{N_a}$$





	Keyword 1	Keyword 2	Keyword 3	...	Keyword N
Date 1	0.42	0.00	0.07		0.23
Date 2	0.00	0.11	0.18		0.04
...					
Date J	0.13	0.29	0.00		0.00

Chapter 7: Sequential Data Models



$$\log p(O | \lambda) = - \sum_{j=0}^{T-1} \log \left(\frac{1}{\sum_{i=0}^{N-1} \hat{\alpha}_t(i)} \right)$$

$$\hat{\beta}_{T-1}(i) = \beta_{T-1}(i) / \sum_{j=0}^{N-1} \beta_{T-1}(j)$$

$$\beta_t(i) = \sum_{j=0}^{N-1} \beta_{t-1}(j) \cdot a_{ij} \cdot b_j(O_{t+1}) \quad c_t = 1 / \sum_{j=0}^{N-1} \beta_t(j) \quad \tilde{\beta}_t(i) = \beta_t(i) \cdot c_t$$

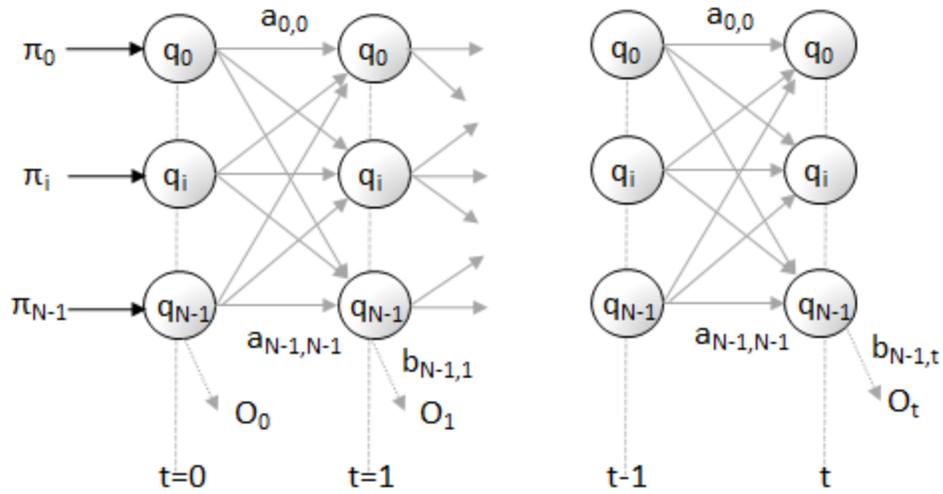
$$\gamma_t(i, j) = p(q_t = S_i, q_{t+1} = S_j | O, \lambda)$$

$$\gamma_t(i, j) = \frac{\alpha_t(i) a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)}{\sum_{j=0}^{N-1} \alpha_t(i) \beta_t(i)}$$

$$\hat{\pi}_t = \gamma_0(i) \quad \gamma_t(i) = \sum_{j=0}^{N-1} \gamma_t(i, j)$$

$$\hat{a}_{ij} = \frac{\sum_{t=0}^{T-1} [\gamma_t(i, j)]}{\sum_{t=0}^{T-1} \gamma_t(i)}$$

$$\hat{b}_{ij} = \frac{\sum_{t=0}^{O_j} \gamma_t(i)}{\sum_{t=0}^{T-1} \gamma_t(i)}$$

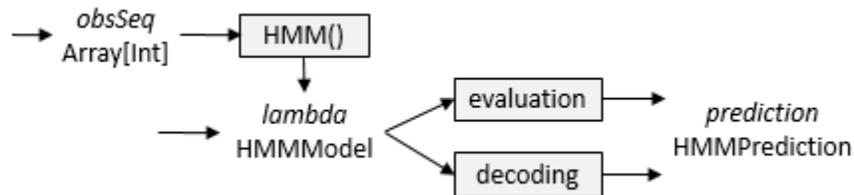


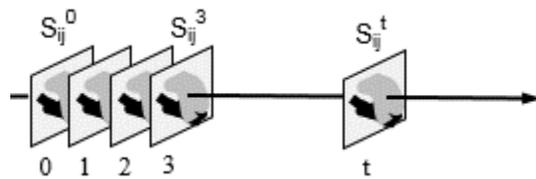
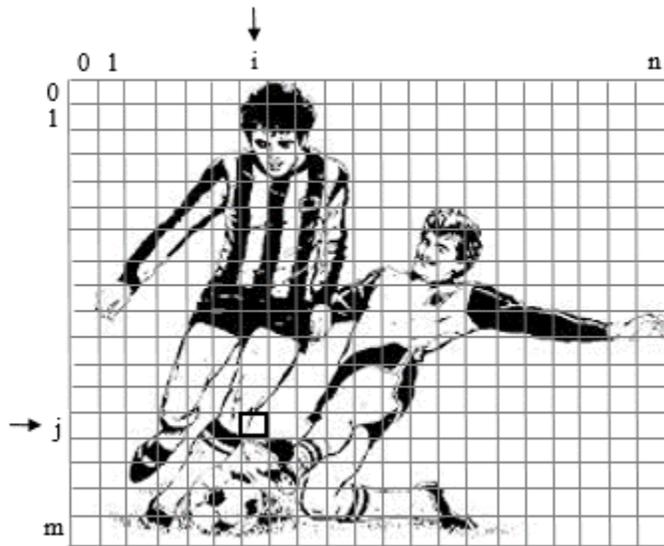
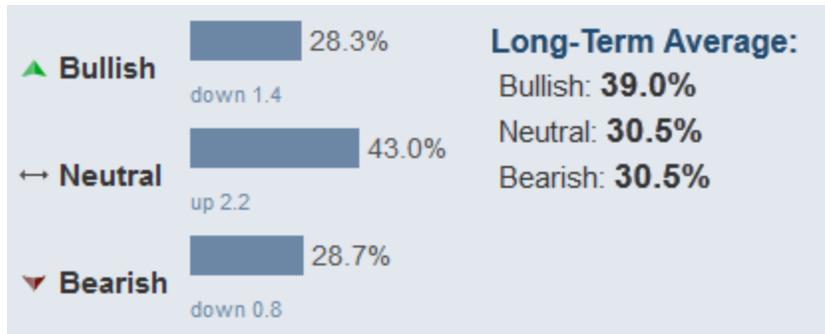
$$\delta_t(i) = \max_{q_{0:T-1}} p(q_{0:T-1} = S_i, O_{0:T-1} | \lambda)$$

$$\delta_0(i) = \pi_i b_i(O_0) \psi_0(i) = 0 \forall i$$

$$\delta_t(j) = \max_i (\delta_{t-1}(i) \cdot a_{ij} \cdot b_j(O_t)) \quad \psi_t(j) = \arg \max_i (\delta_{t-1}(i) \cdot a_{ij})$$

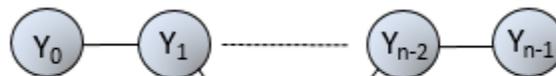
$$q_t^* = \psi_{t+1}(q_{t+1}^*) \quad q_T^* = \arg \max_i \delta_T(i)$$





Sequences of labels (type of interaction)

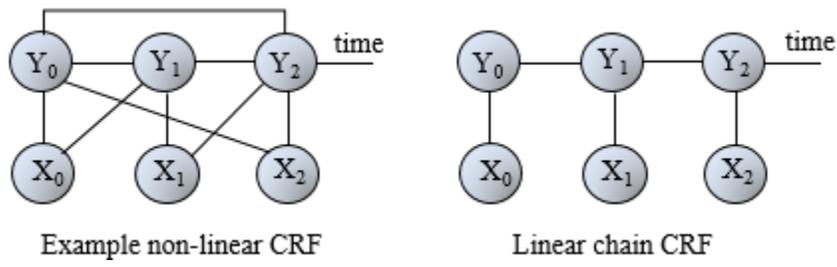
$Y = \{\text{legitimate, in-doubt, inappropriate, dangerous}\}$



Features

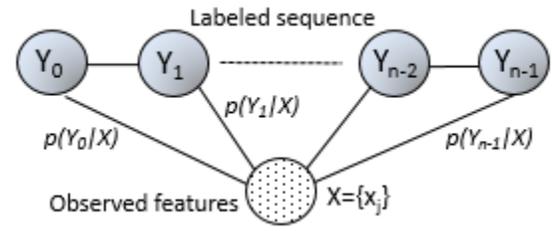
$X = \{\text{color, texture, edge, ...}\}$





Example non-linear CRF

Linear chain CRF

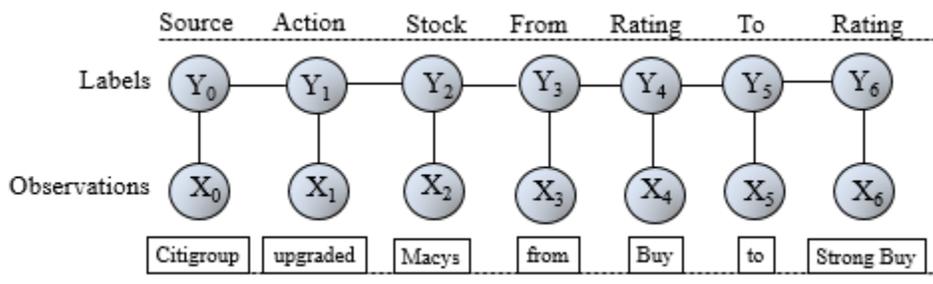


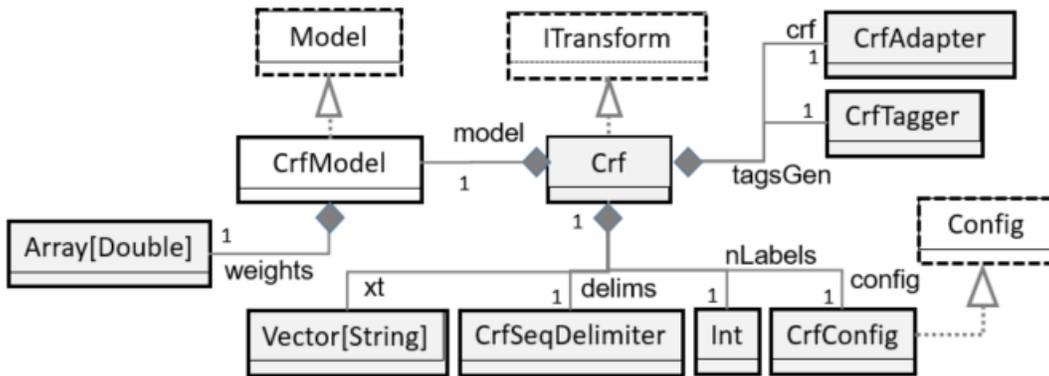
$$\log f_i(y_{i-1}, y_i, x, i) = w_c + \sum_{i=0}^{K-1} w_i t_i(y_{i-1}, y_i, x, i) + \sum_{j=0}^{K-1} \mu_j s_j(y_i, x, i)$$

$$t_i(y_{i-1}, y_i, x, i) = I(y_{i-1} = l_1) \cdot I(y_i = l_2) \cdot I(x = 0)$$

$$F_i(y, x) = \sum_{j=0}^{K-1} f_j(y_{j-1}, y_j, x, i) \quad \log p(x, \lambda) \propto \sum_{j=0}^{K-1} w_j F_j(x, y)$$

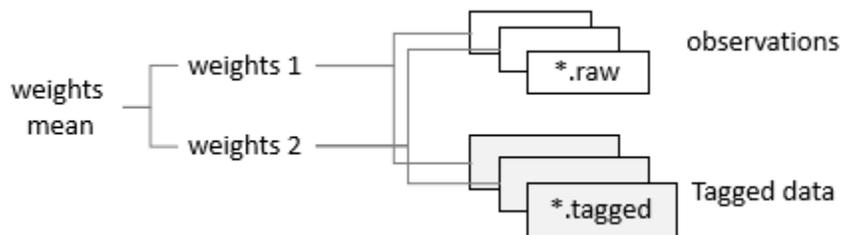
$$p(y|x, w) = \frac{1}{Z(x)} e^{\sum_{j=0}^{K-1} w_j F_j(x, y)} \quad z(x) = \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} w_j F_j(x, y)$$

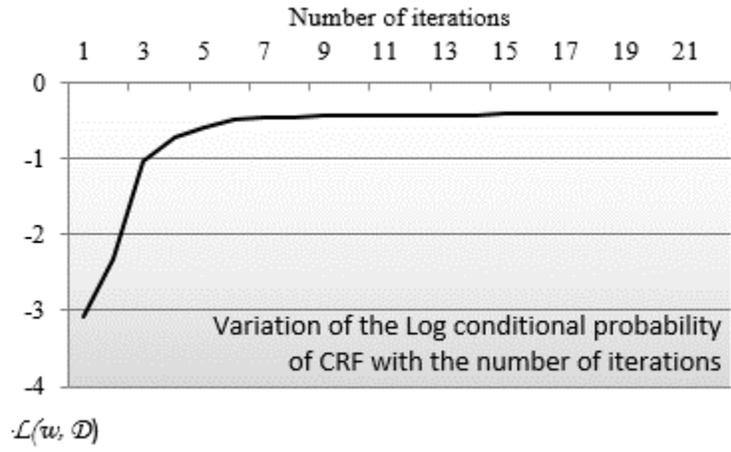




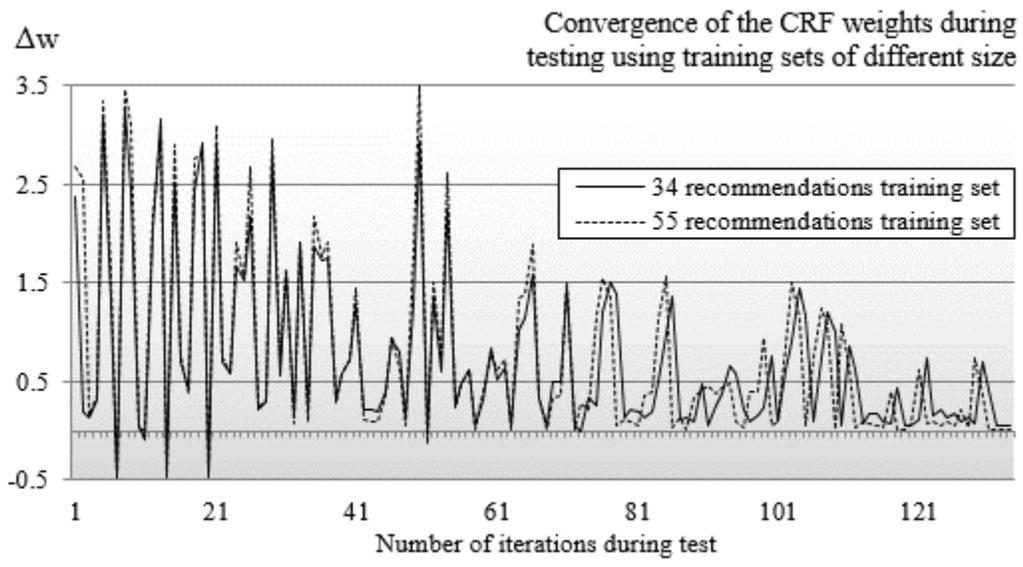
$$\mathcal{L}(w, D) = -\sum_{i=0}^{n-1} \log p(y_i | x_i, w)$$

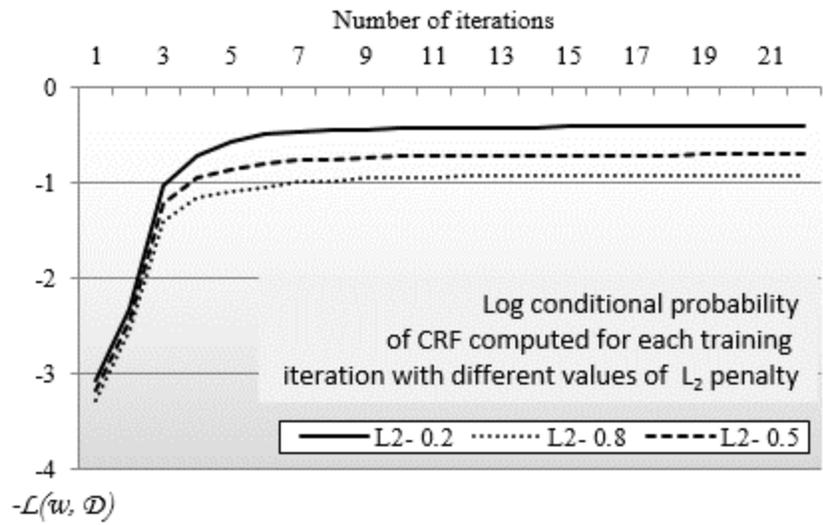
$$w^* = \arg \max_{\lambda} \left[\mathcal{L}(w, D) + \lambda \|w\|^2 \right] \quad \lambda = \frac{1}{2\sigma^2}$$





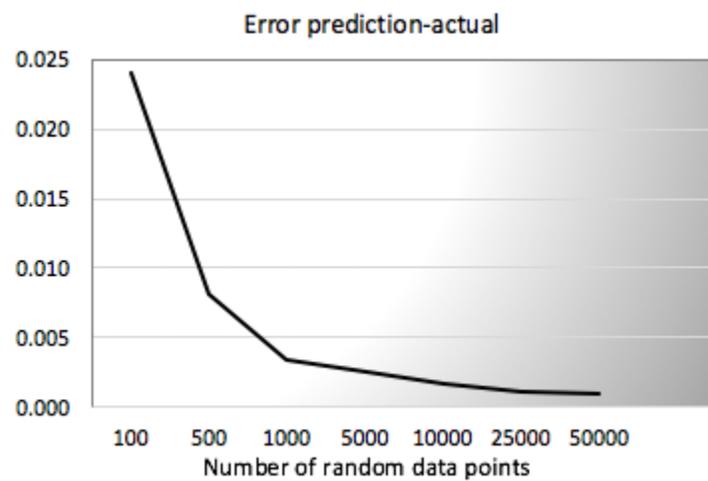
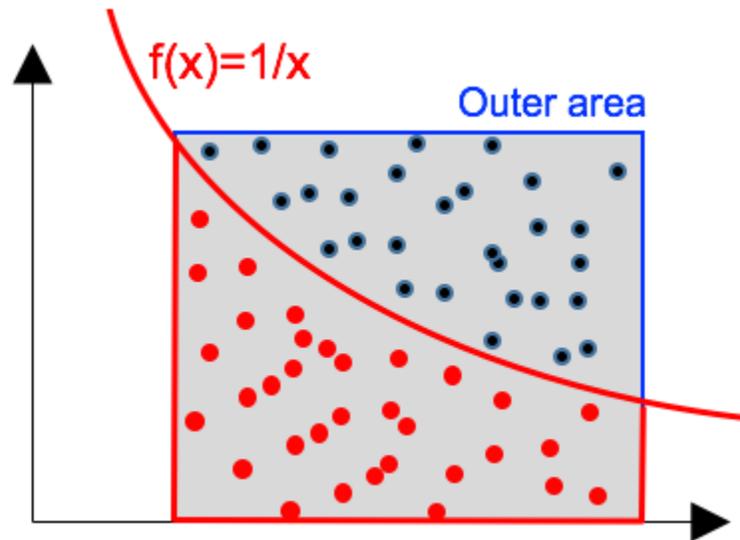
$$\Delta w = \sum_{i=0}^{D-1} (w_i^{t+1} - w_i^t)$$





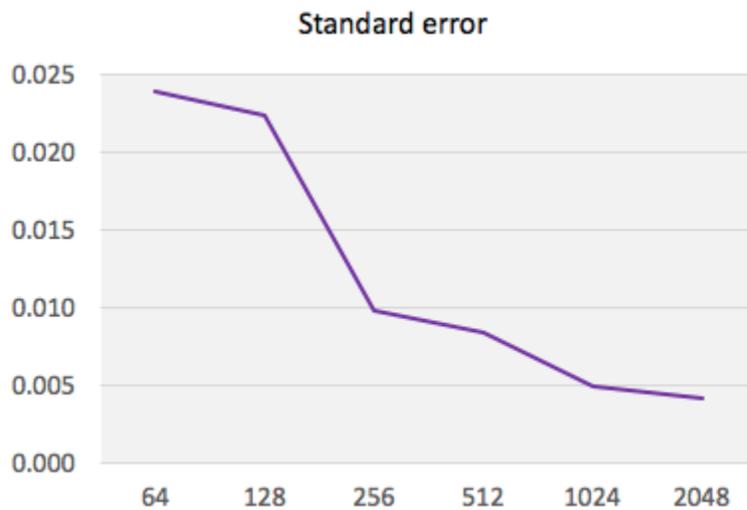
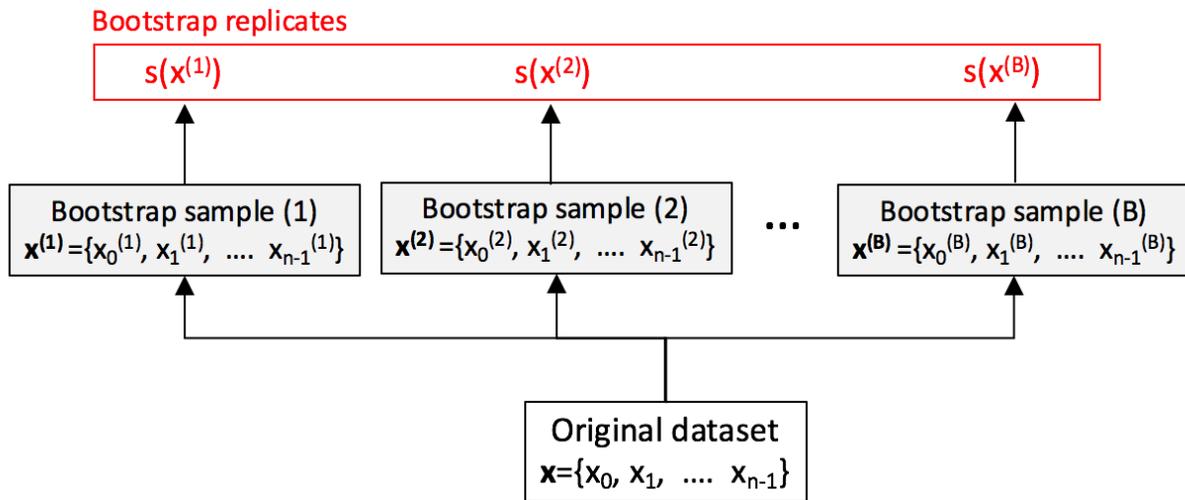
Chapter 8: Monte Carlo Inference

$$\sqrt{-2 \log(u_1)} \cdot \sin(2\pi u_2) \quad \sqrt{-2 \log(u_1)} \cdot \cos(2\pi u_2)$$



$$x^{(i)} = \{x_j^{(i)}\} \quad \hat{\theta}^{(i)} = s(x^{(i)})$$

$$\hat{s} = \sqrt{\frac{1}{B-1} \sum_{j=0}^{B-1} (\hat{\theta}_j^{(i)} - \bar{\theta})^2} \quad \hat{\theta} = \frac{1}{B} \sum_{j=0}^{B-1} \hat{\theta}_j^{(i)}$$



$$\alpha = \frac{\hat{p}(s')q(s^t | s')}{\hat{p}(s^t)q(s' | s^t)}$$

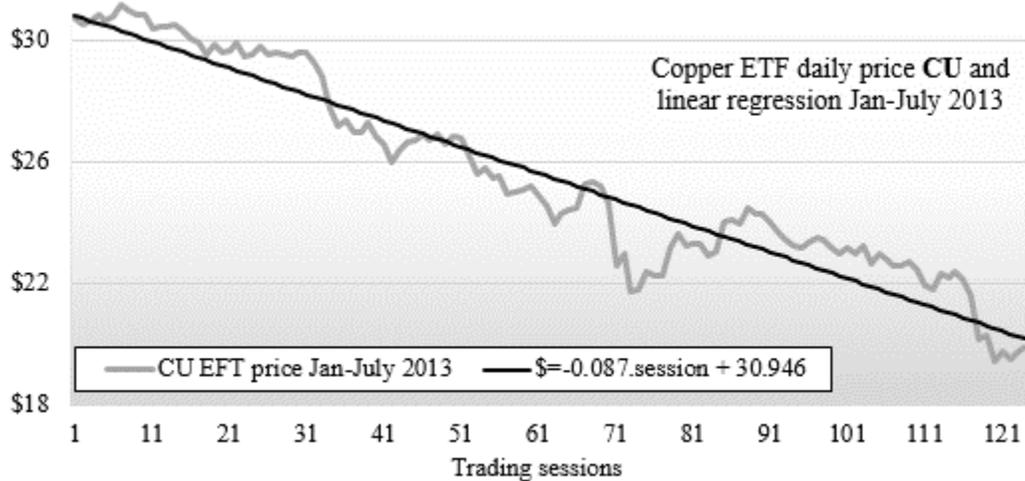
$$s^{t+1} = \begin{cases} s' & \text{if } u < \min(1, \alpha) \\ s^t & \text{if } u \geq \min(1, \alpha) \end{cases}$$

$$\log \alpha = \log(\hat{p}(s')) - \log(\hat{p}(s^t)) + \log(q(s^t | s')) - \log(q(s' | s^t))$$

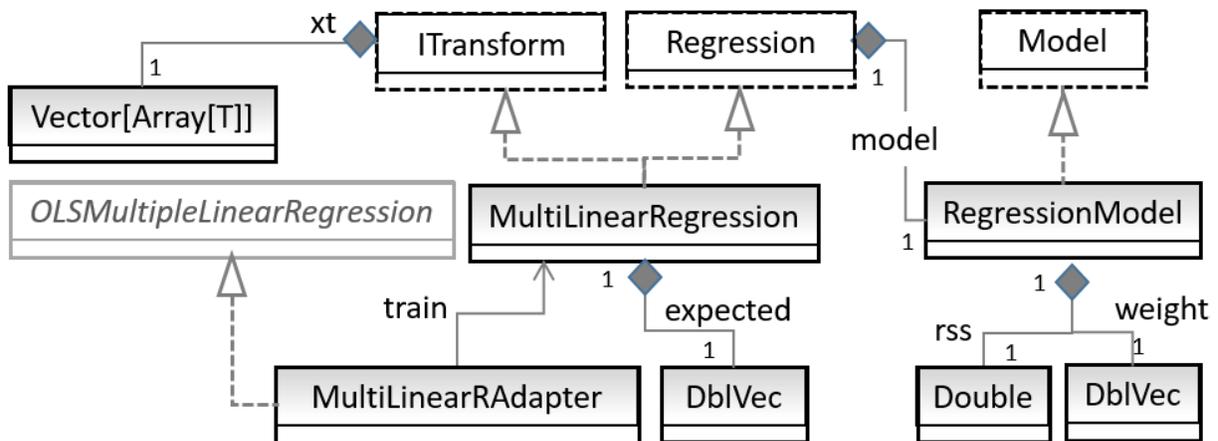
$$s^{t+1} = \begin{cases} s' & \text{if } u < e^\alpha \\ s^t & \text{if } u \geq e^\alpha \end{cases}$$

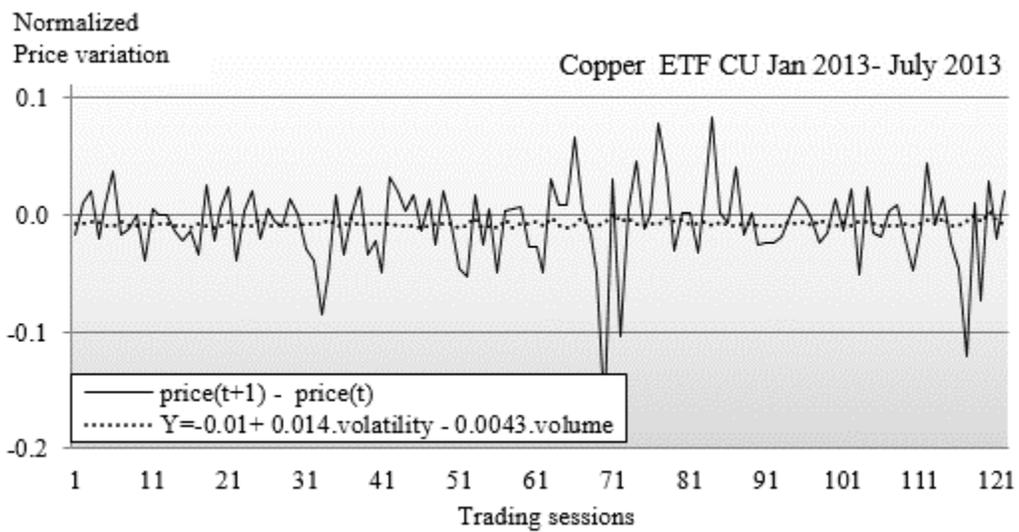
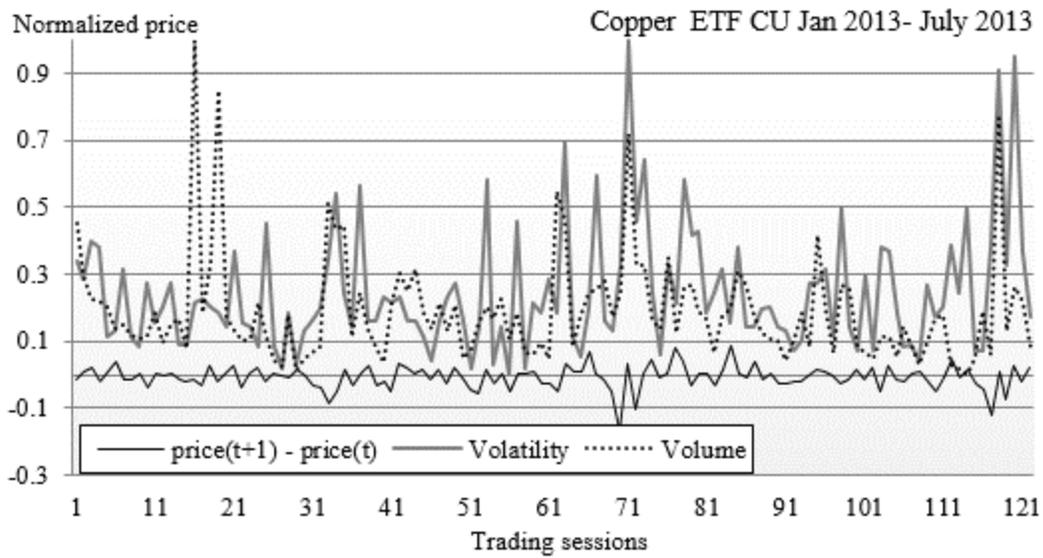
Chapter 9: Regression and Regularization

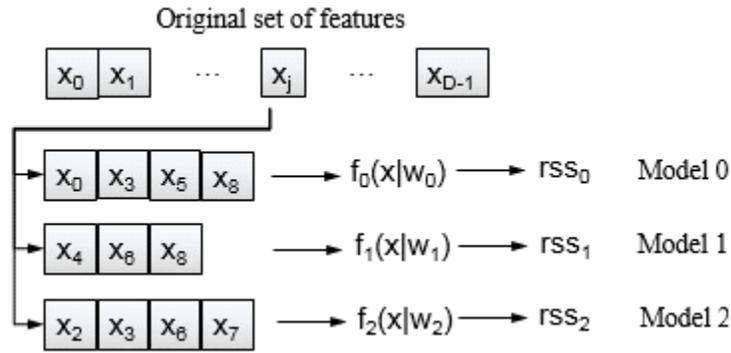
$$\tilde{w} = \arg \min_{w,r} \left\{ \sum_{j=0}^{N-1} (y_j - f(x_j|w))^2 \right\} \quad f(x|w) = w_0 + w_1 x$$



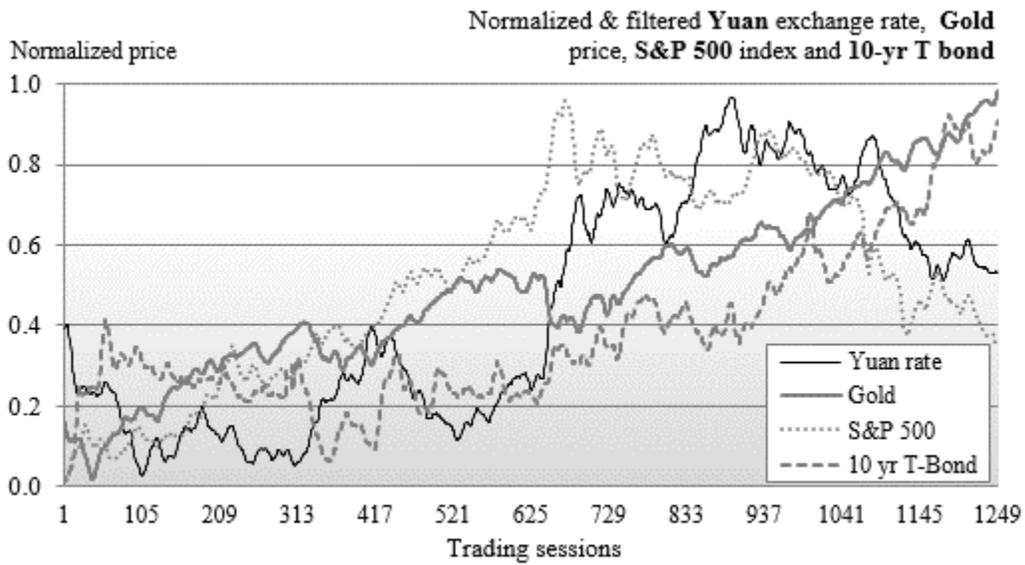
$$\tilde{w} = \arg \min_{w,r} \left\{ \sum_{j=0}^{N-1} (y_j - f(x_j|w))^2 \right\} \quad f(x|w) = w_0 + \sum_1^{D-1} w_d x_d$$







$$\tilde{f} = \arg \min_{f_j} \left\{ \sum_{i=0}^{n-1} (y_i - f_j(x|w)) \right\} \quad f_j(x|w) = w_{j0} + \sum_{d=1}^{D_j-1} w_{jd} x_d$$



$$r^2 = 1 - \frac{RSS}{TSS} \quad TSS = \sum_{i=0}^{n-1} (y_i - \bar{f}(x|w))^2 \quad \bar{f} = \sum_f f_j$$

CNY = f(SPY, GLD, TLT)
 $0.16089780923264457 + -0.21189413823325406.x1 + 0.26299169969099795.x2 + 0.3556562652009136.x3$
RSS: 3.681353535940423

CNY = f(SPY, TLT)
 $0.2039015515038045 + -0.03796334296279046.x1 + 0.26219728078589966.x2$
RSS: 3.8589613138639227

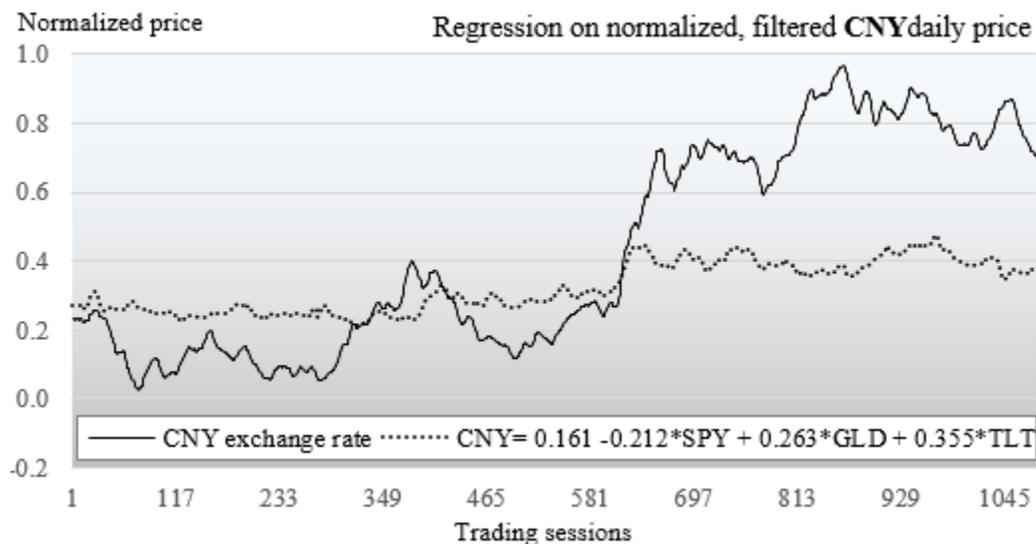
CNY = f(GLD, TLT)
 $0.19290917330324198 + 0.015507174710195552.x1 + 0.2204800144601237.x2$
RSS: 3.8849688539396317

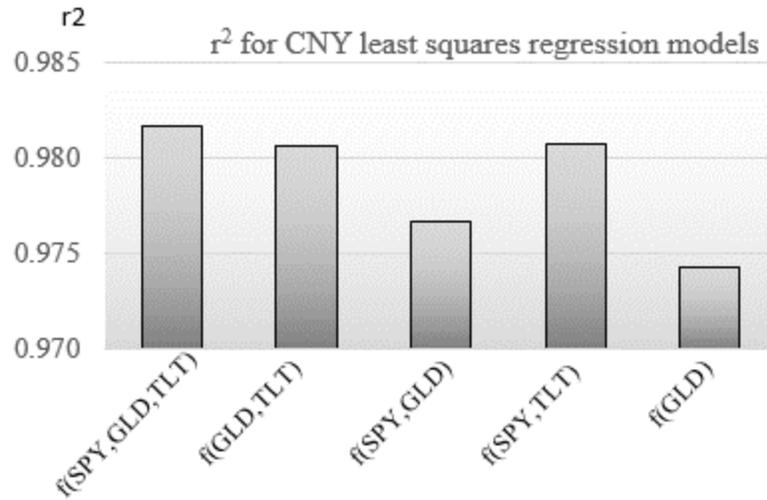
CNY = f(SPY, GLD)
 $0.22242202699107552 + 0.17842973203100937.x1 + -0.12099602178260839.x2$
RSS: 4.6681933948464645

CNY = f(SPY)
 $0.20238901847764892 + 0.1251591898720694.x1$
RSS: 4.7291908591838405

CNY = f(TLT)
 $0.19724352413716711 + 0.22501420632545652.x1$
RSS: 3.8876824376753705

CNY = f(GLD)
 $0.198195293931846 + 0.16413676262473123.x1$
RSS: 5.149661975952835





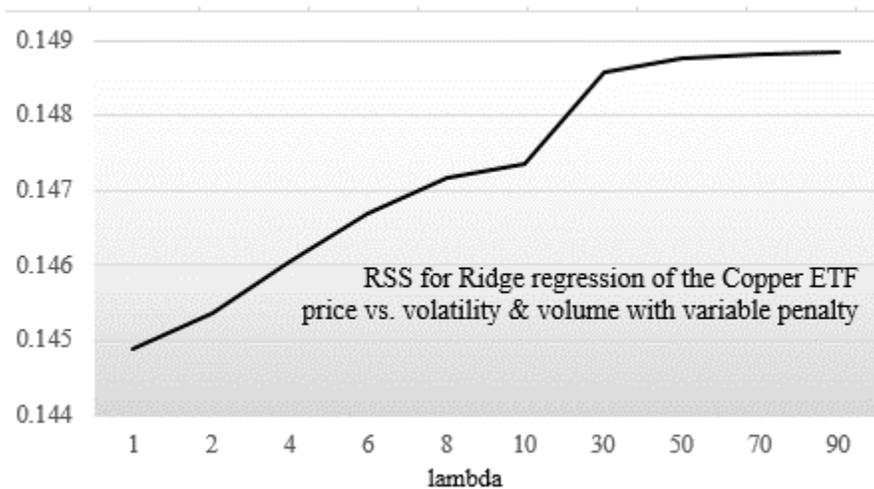
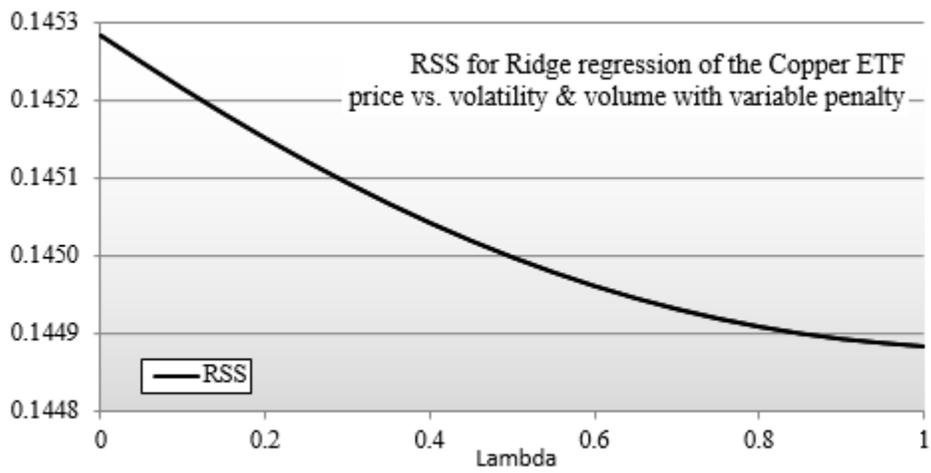
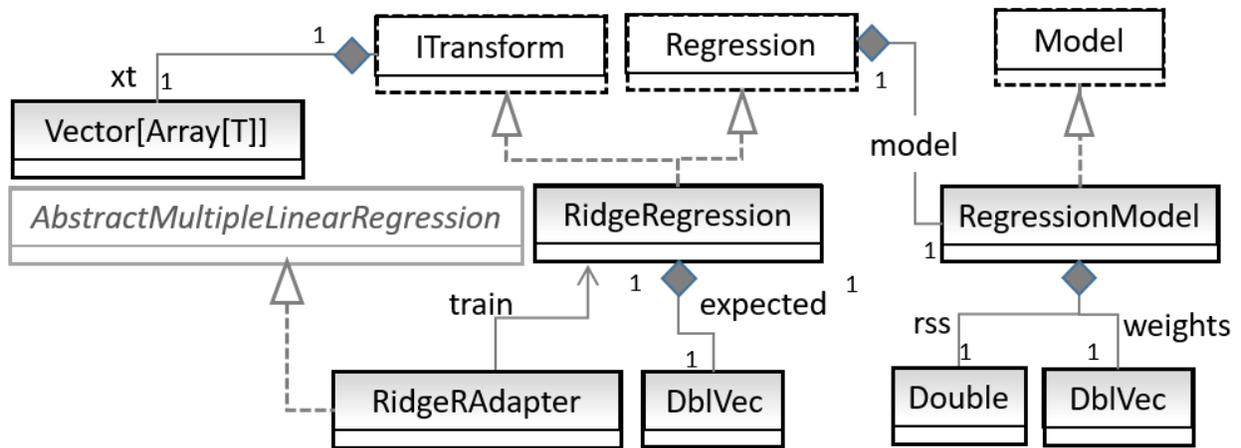
$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}_d} \left\{ \sum_{i=0}^{n-1} (y_i - f(\mathbf{x}_i | \mathbf{w}))^2 + \lambda J(\mathbf{w}) \right\}$$

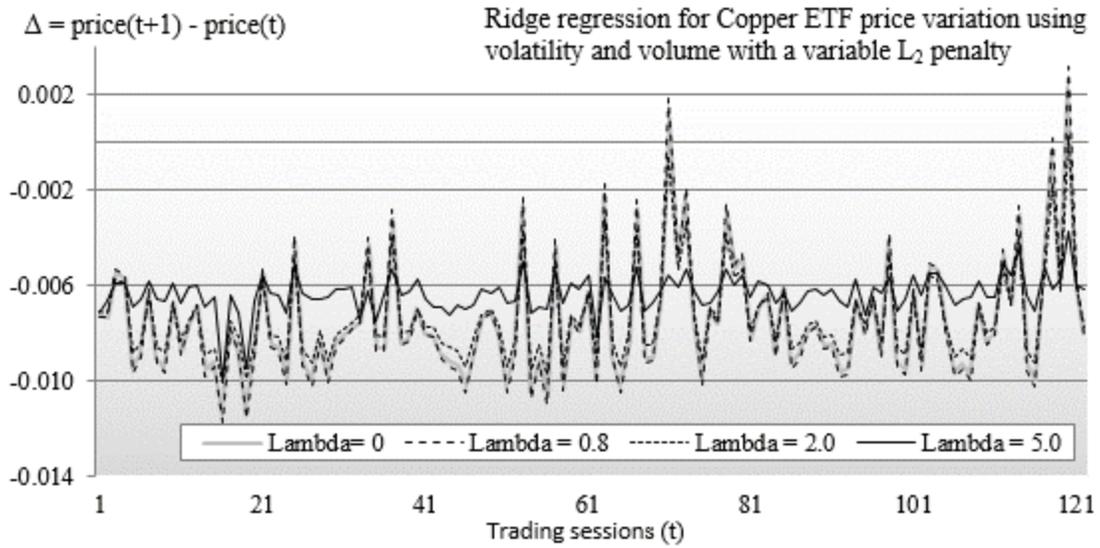
$$J_{pq}(\mathbf{w}) = \|\mathbf{w}\|_p^q = \left[\sum_{d=1}^{D-1} |w_d|^p \right]^{q/p}$$

$$\tilde{\mathbf{w}}_{ridge} = \arg \min_{\mathbf{w}} \left\{ \sum_{j=0}^{N-1} (y - w_0 - \mathbf{w}^T \mathbf{x})^2 + \lambda \|\mathbf{w}\|_2^2 \right\} \quad \|\mathbf{w}\|_2^2 = \sum_1^{D-1} w_d^2$$

$$\{X^T X - \lambda I\} \cdot \hat{\mathbf{w}}_{Ridge} = X^T y$$

$$\{X^T X - \lambda I\} = Q \begin{bmatrix} R \\ 0 \end{bmatrix} \quad \mathbf{w}_{Ridge} = Q^T y \begin{bmatrix} R \\ 0 \end{bmatrix}^{-1}$$



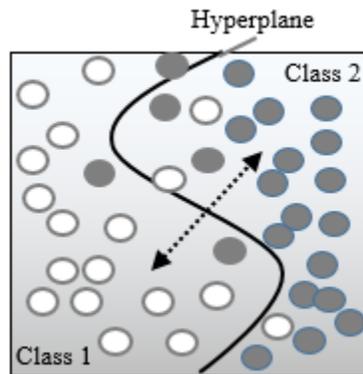
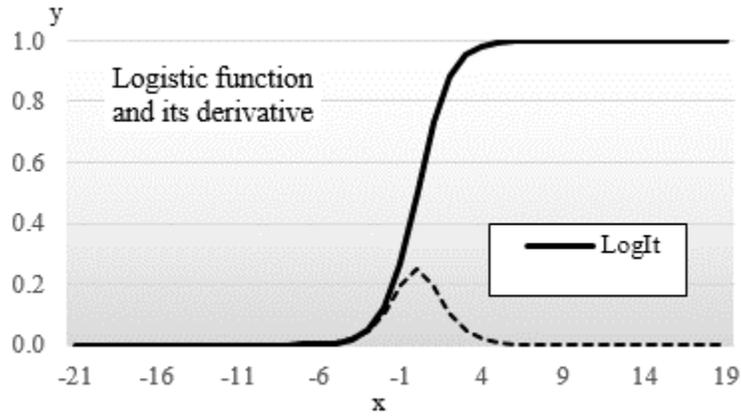


$$L(w) = \sum_{i=0}^{n-1} r_i(w)^2 \quad r_i(w) = y_i - f(x_i|w)$$

$$\sum_{i=0}^{n-1} r_i(w) J_{id}(w) = 0 \quad J_{id}(w) = -\frac{\partial r_i(w)}{\partial w_d}$$

$$f(x_i|w) - f(x_i|w^{(k)}) \sim \sum_{jd=0}^{D-1} \frac{\partial f(x_i|w^{(k)})}{\partial w_d} (w - w^{(k)})$$

$$f(x) = \frac{1}{(1 - e^{-x})} \quad \frac{df}{dx} = f(x)(1 - f(x))$$



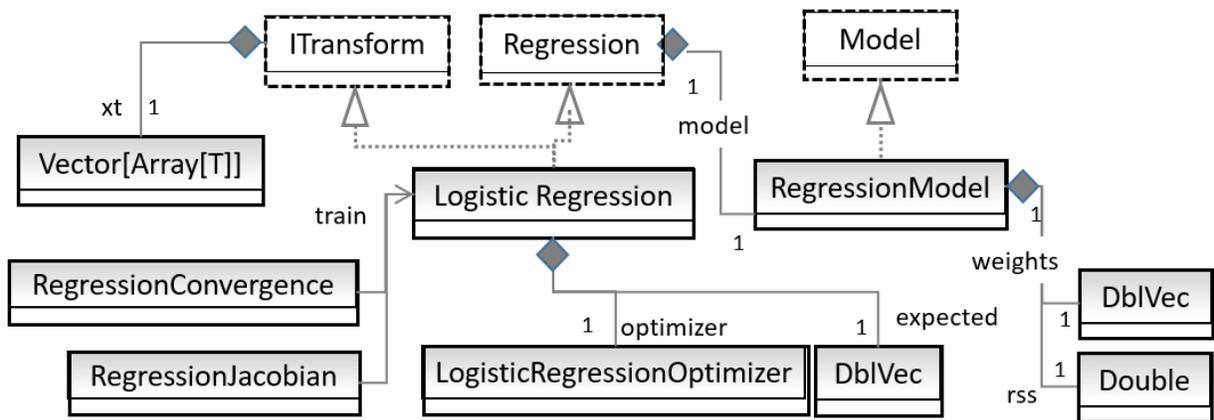
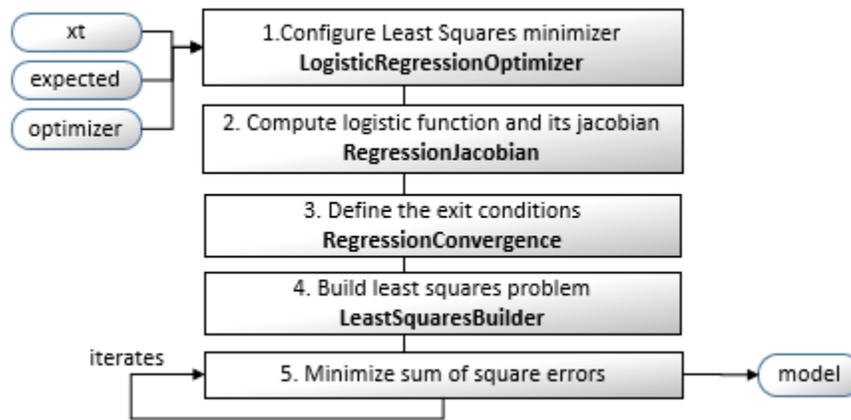
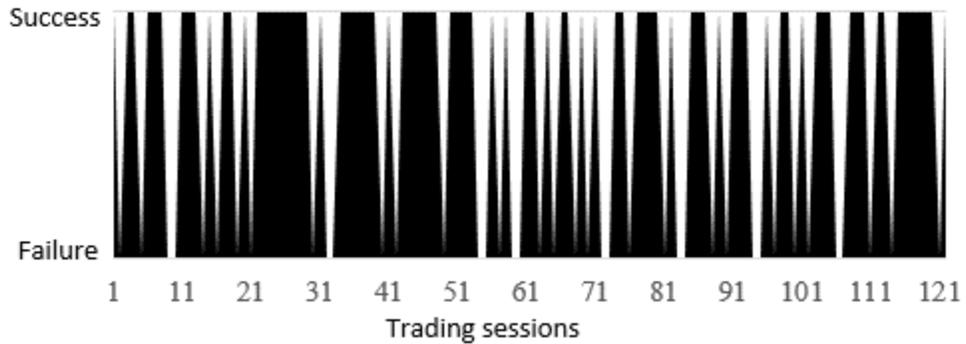
$$L(w) = \sum_{i=0}^{N-1} \log p(x_i | w)$$

$$x_i = \{1, x_{i0}, \dots, x_{id-1}\} \quad p(x_i | w) = \frac{1}{1 + e^{-w^T x_i}} \quad w^T x_i = \sum_{j=0}^d w_j x_{ij}$$

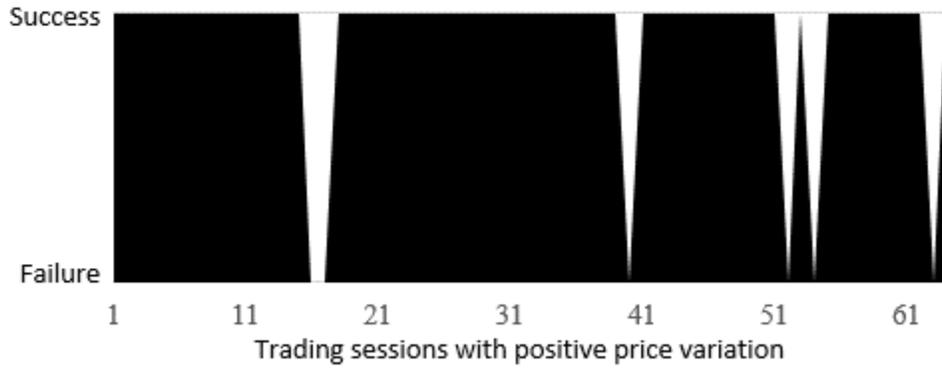
$$sse(w) = \frac{1}{2} \sum_{i=0}^{N-1} \left\{ y_i - \log(1 + e^{-w^T x_i}) \right\}^2 \quad y \in \{0, 1\}$$

$$\frac{\partial L(\tilde{w})}{\partial w_j} = \sum_{i=0}^{N-1} x_{ij} \left(y_i - \frac{1}{1 + e^{-\tilde{w}^T x_i}} \right) = 0$$

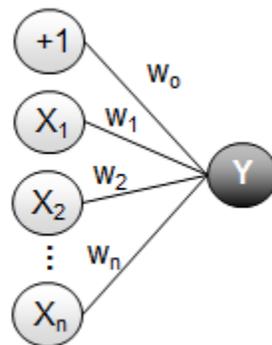
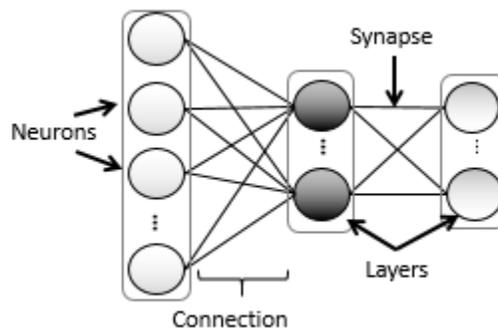
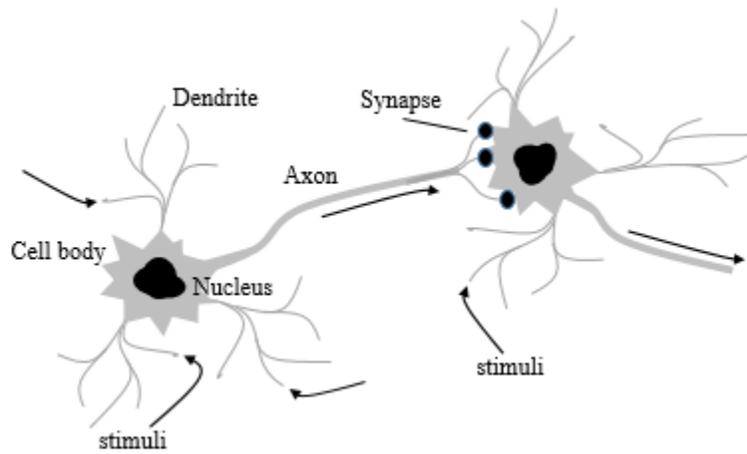
Accuracy of prediction of the direction of the price variation of Copper ETF given its volatility and trading volume



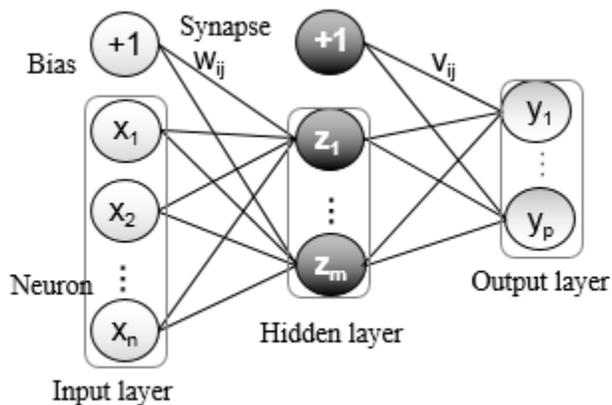
Accuracy of prediction of the positive price variation of Copper ETF given its volatility and trading volume



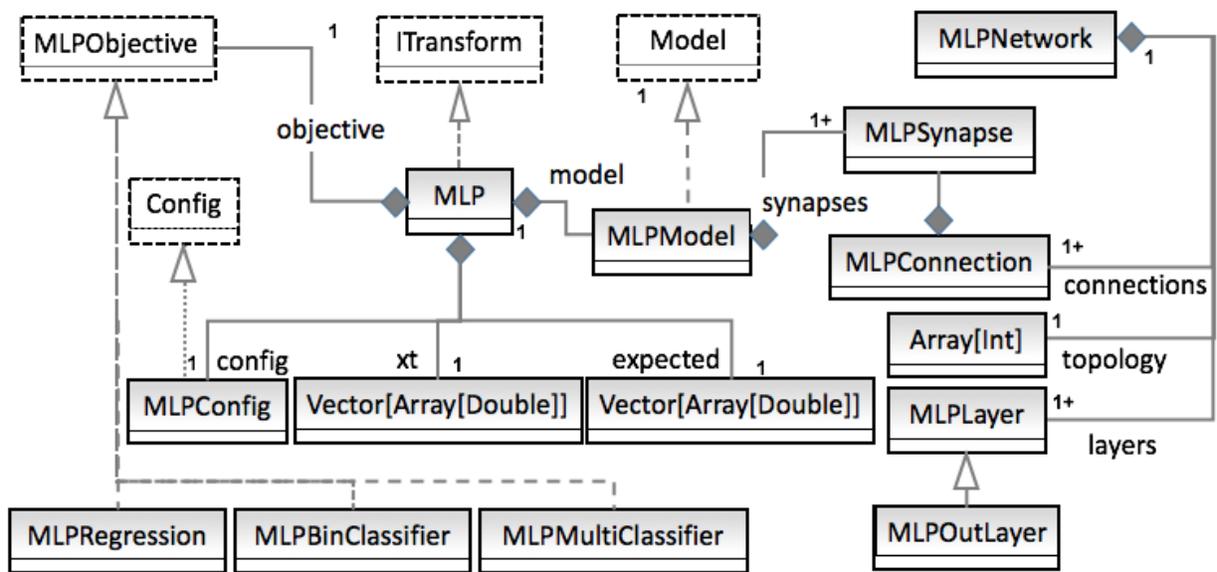
Chapter 10: Multilayer Perceptron

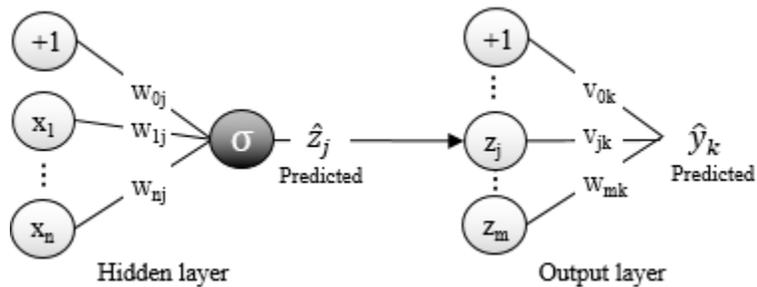
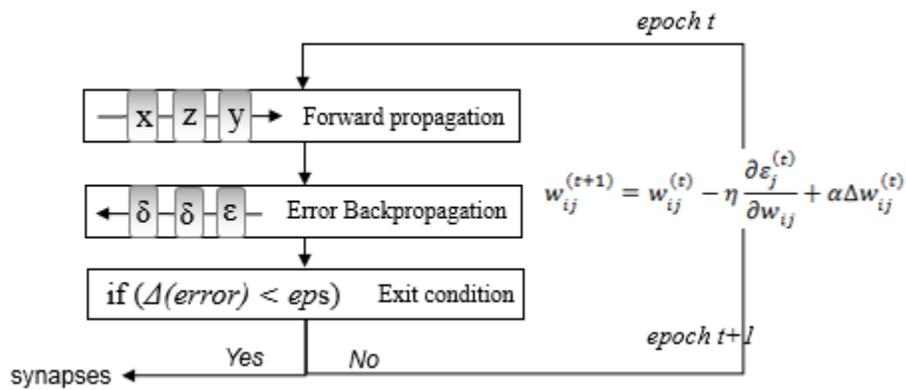
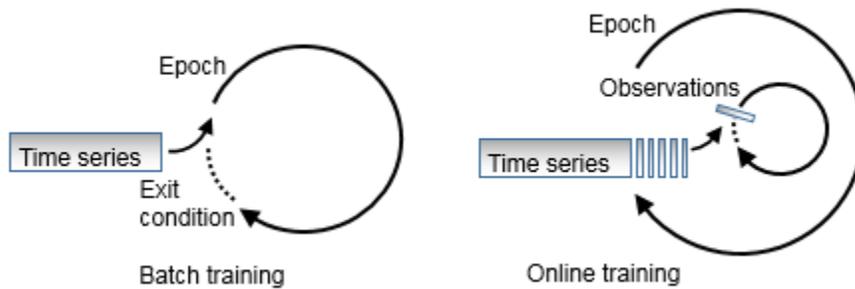
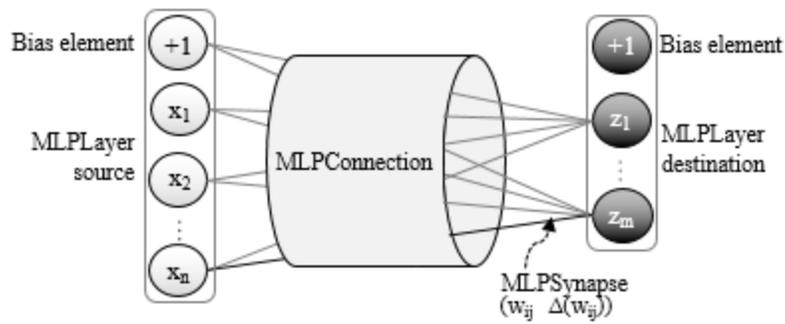


$$y = \sigma(w_0 + w^T x) = \frac{1}{1 + e^{-(w_0 + w^T x)}}$$



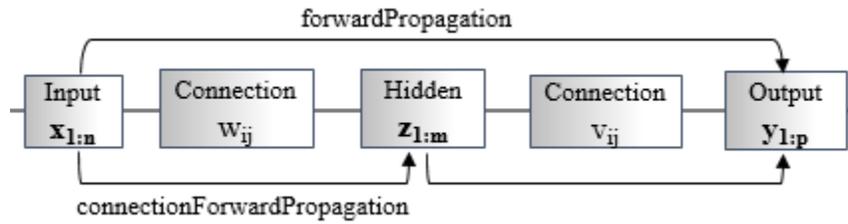
$$\hat{y} = h \left(w_0 + \sum_{i=1}^n w_i x_i \right) = h \left(w_0 + w^T x \right)$$





$$\tilde{y}_k = v_{0j} + \sum_{j=1}^m v_{kj} z_j$$

$$\tilde{z}_j = \sigma \left(w_{0j} + \sum_{i=1}^m w_{ij} x_i \right) = \frac{1}{1 + e^{-w_{0j} - \sum_{i=1}^m w_{ij} x_i}}$$



$$\mathcal{E} = \frac{1}{2} \sum_{i=0}^{n-1} \sum_{j=0}^{k-1} (y_{ij} - \tilde{y}_{ij})^2 \quad \bar{\mathcal{E}} = \frac{\mathcal{E}}{n}$$

$$ce = - \sum_{i=0}^{n-1} \{ y_i \log(\tilde{y}_i) + (1 - y_i) \cdot \log(\tilde{y}_i) \}$$

$$ce = - \sum_{i=0}^{n-1} \sum_{j=0}^{k-1} \tilde{y}_{ij} \log(y_{ij})$$

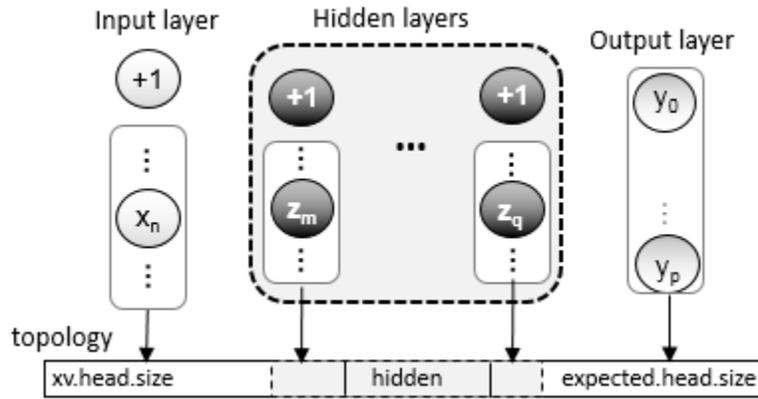
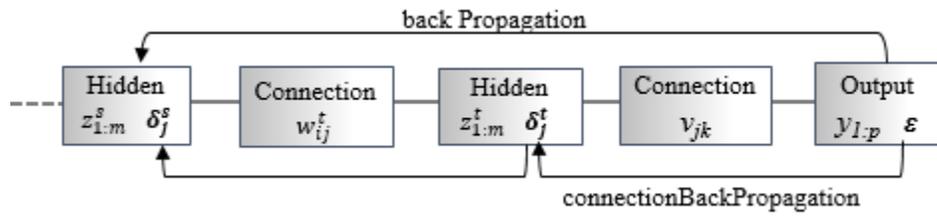
$$\hat{y} = \frac{e^{-\hat{y}_k}}{\sum_i e^{-\hat{y}_i}}$$

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \frac{\partial \mathcal{E}_j^{(t)}}{\partial w_{ij}}$$

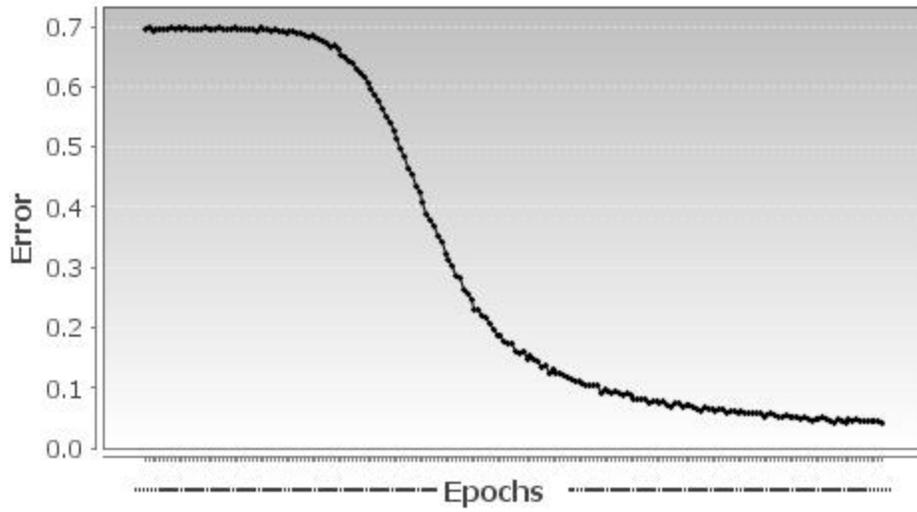
$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \frac{\partial \mathcal{E}_j^{(t)}}{\partial w_{ij}^{(t)}} + \alpha \Delta w_{ij}^{(t)}$$

$$\delta_{ih} = (\tilde{y}_i - y_i) \cdot z_h \quad \Delta v_{ih} = -\delta_{ih}$$

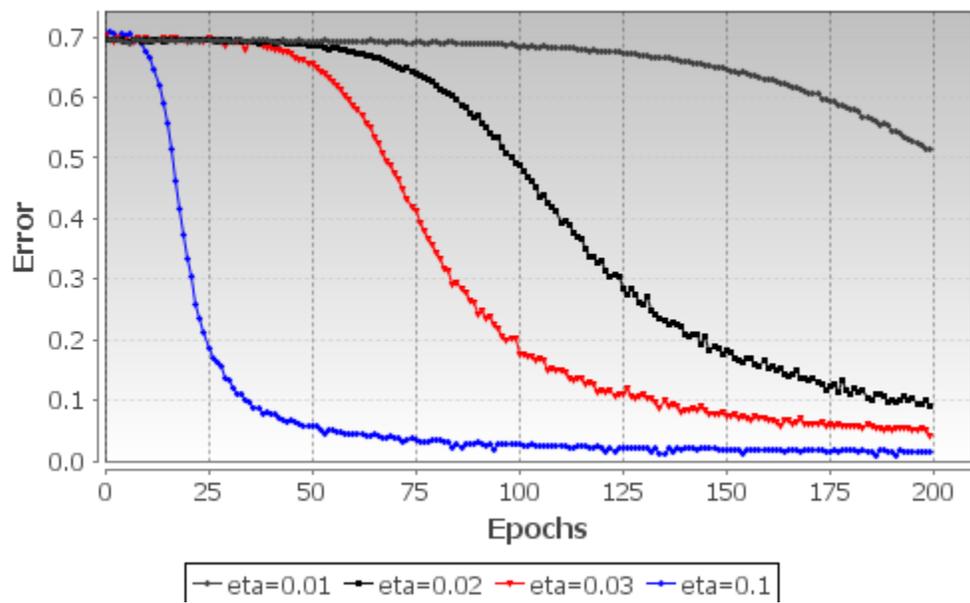
$$\delta_{hi} = \sum_{j=0}^{k-1} \left\{ (\tilde{y}_j - y_j) \cdot v_{jh} \right\} \cdot z_h (1 - z_h) \cdot x_i \quad \Delta w_{hi} = -\eta \delta_{hi}$$



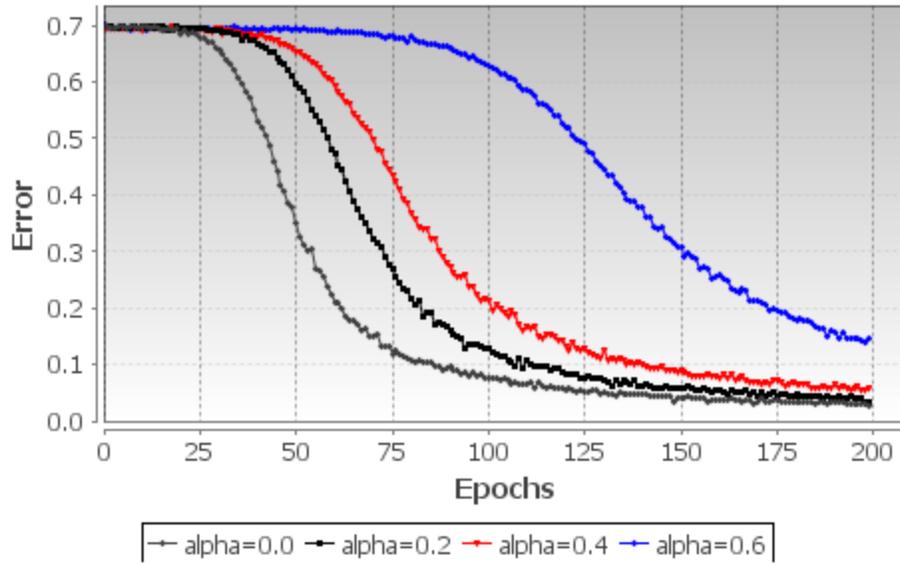
MLP [2-3-1] eta=0.03, alpha=0.3



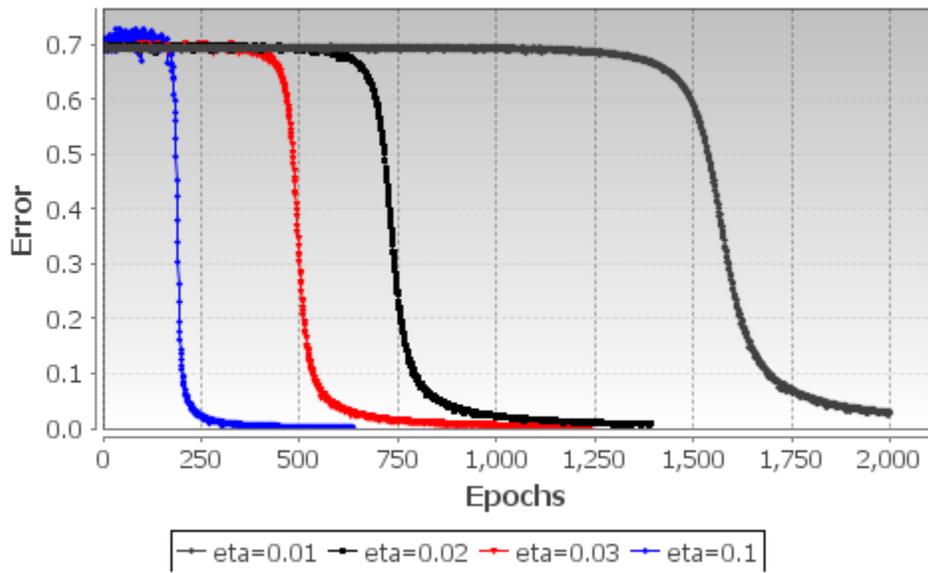
MLP [2-3-1] training - learning rate

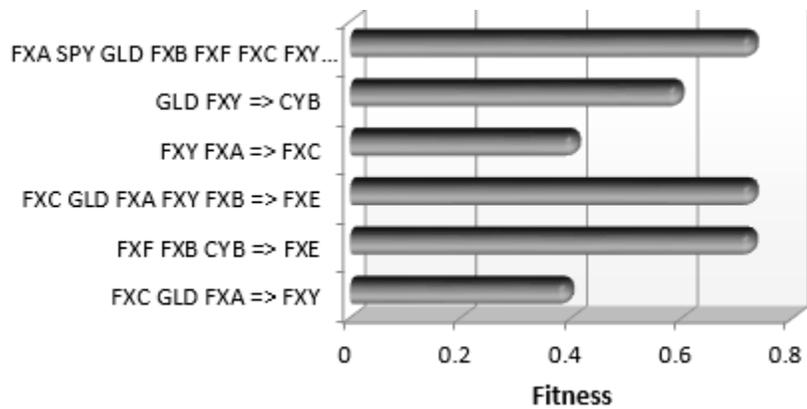
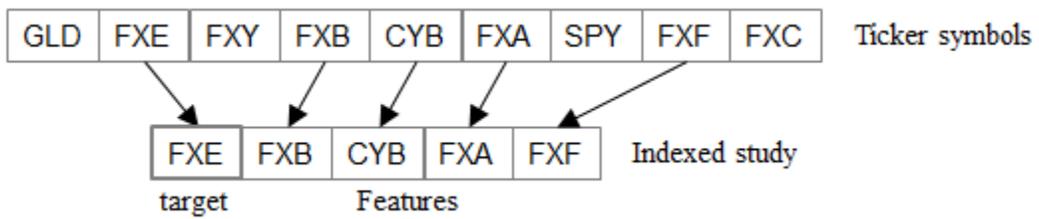
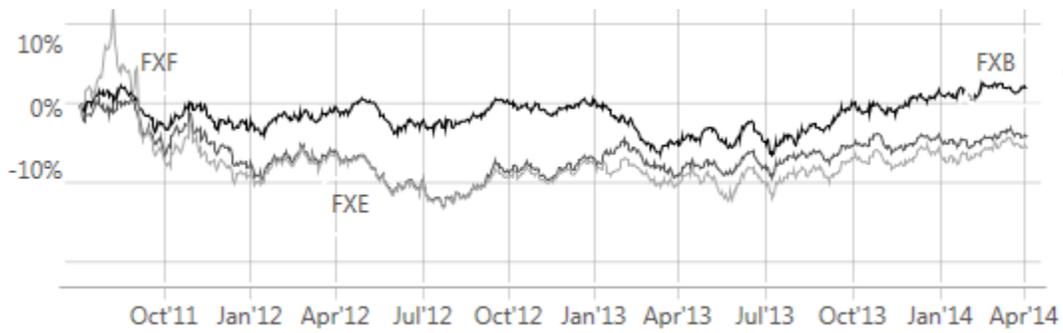


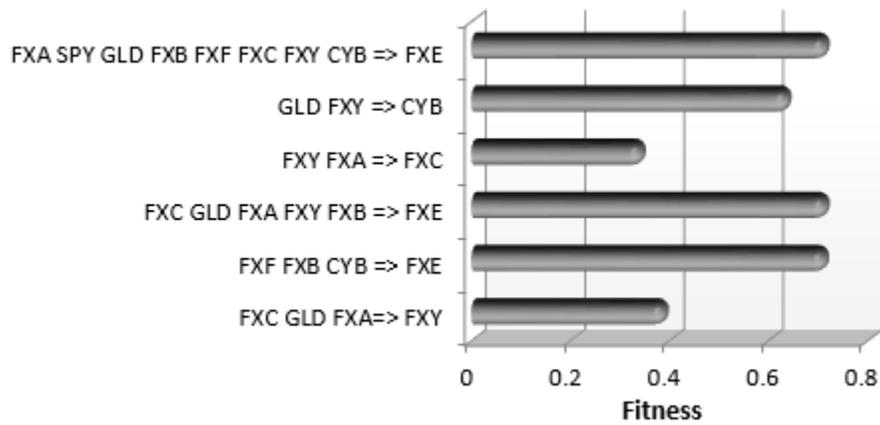
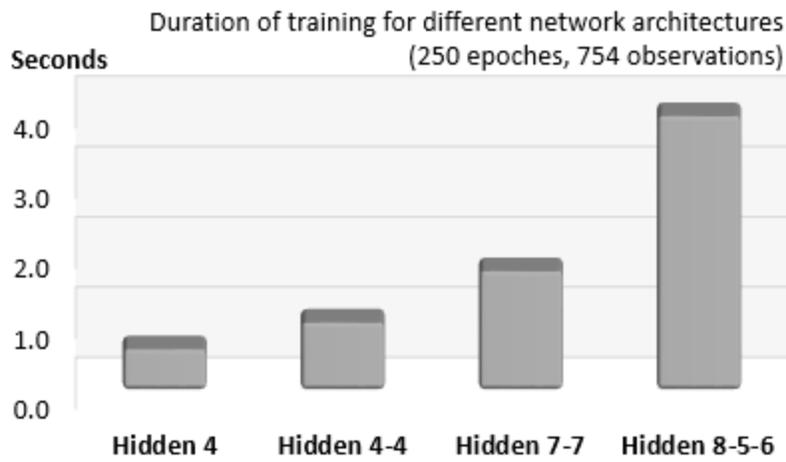
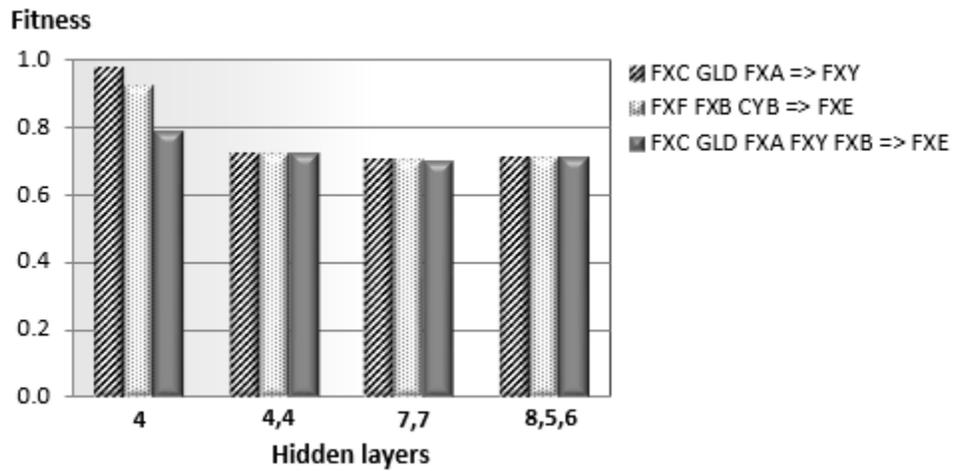
MLP [2-3-1] training - momentum

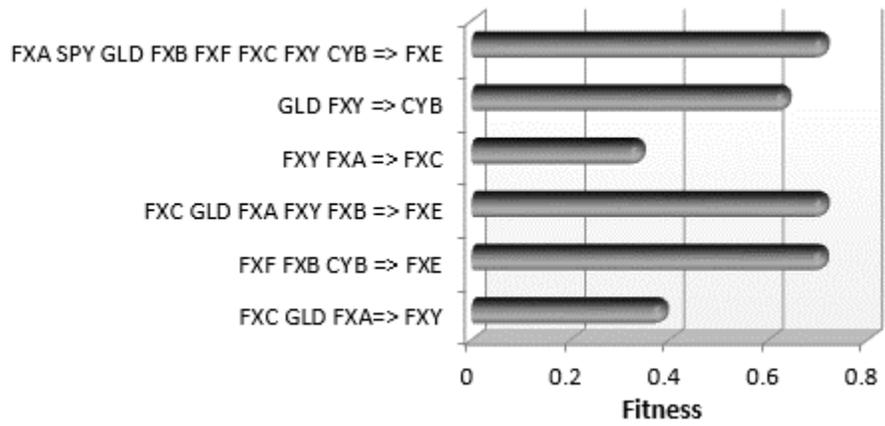


MLP [2-7-3-1] training - learning rate

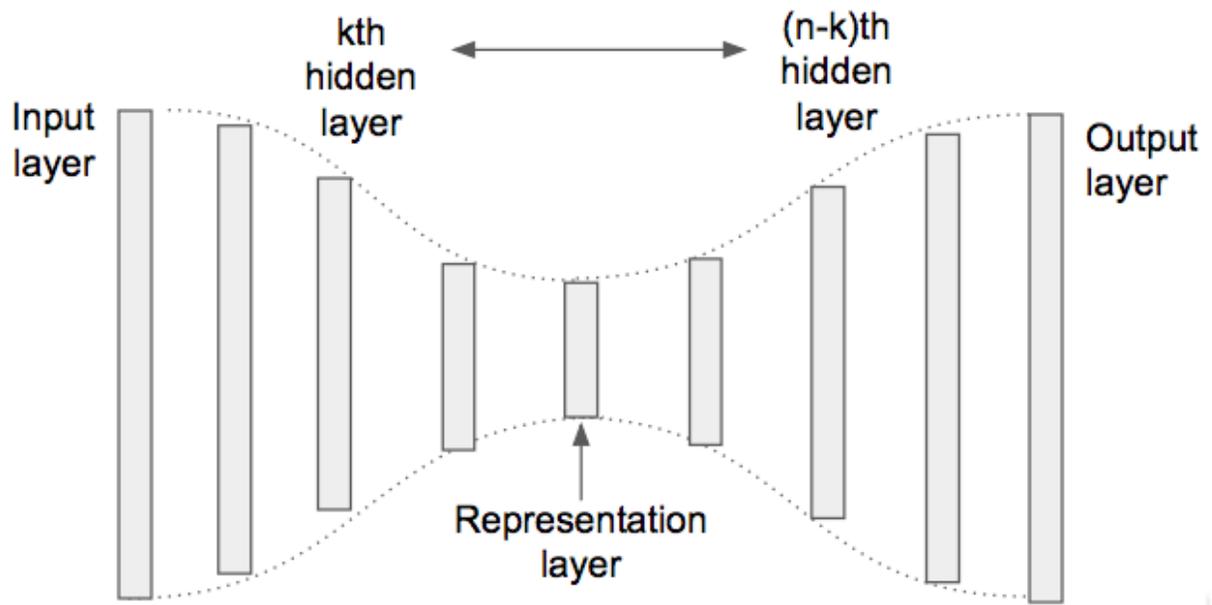








Chapter 11: Deep Learning

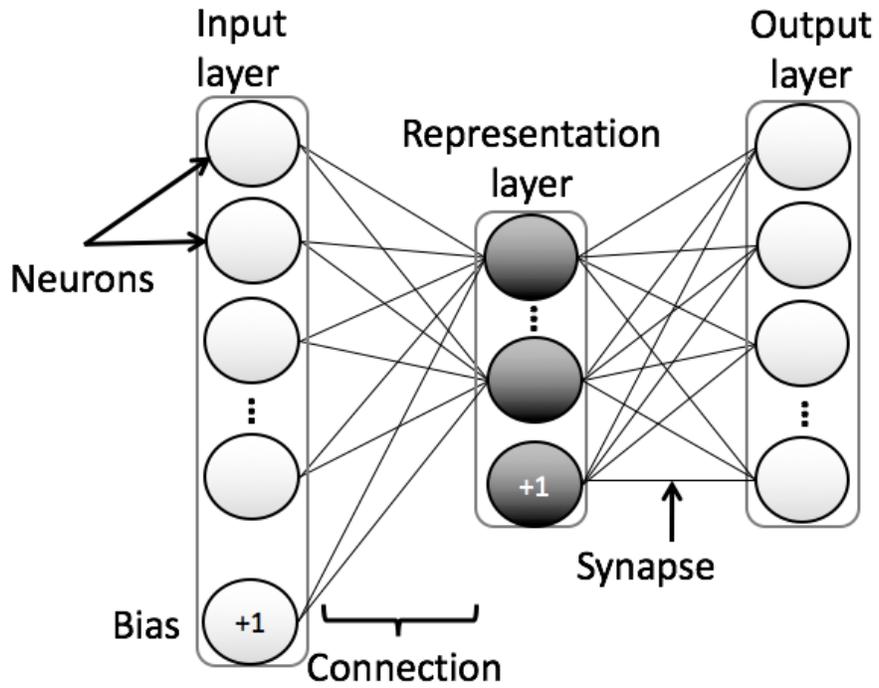


$$x' = \mathcal{O}(\psi(x)) \text{ loss} = \mathcal{L}(x, \mathcal{O}(\psi(x)))$$

$$h = \mathcal{O}(x) = \sigma(w^t \cdot x + b)$$

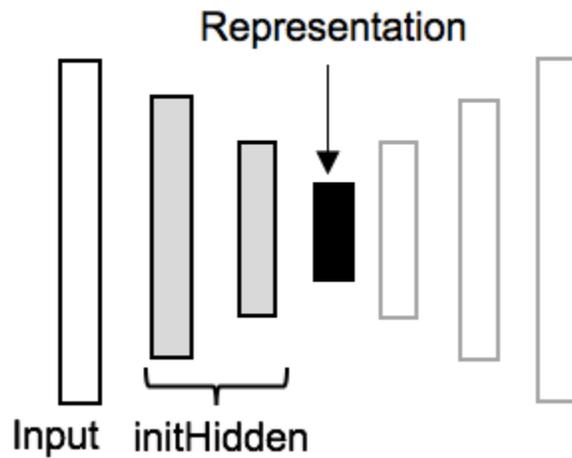
$$x' = \psi(h) = \sigma'(w'' \cdot h + b)$$

$$\text{loss} = \|x - x'\|^2$$



$$\rho'_i = \lambda \cdot f(w_i^T \cdot x + b'_i) + (1 - \lambda) \cdot \rho$$

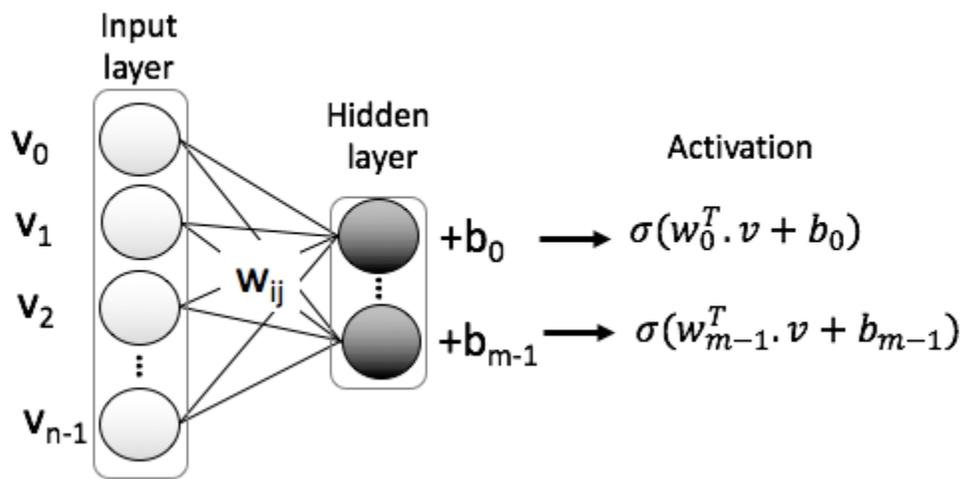
$$b'_i = b_i - \alpha \beta (\rho_i - \rho'_i)$$



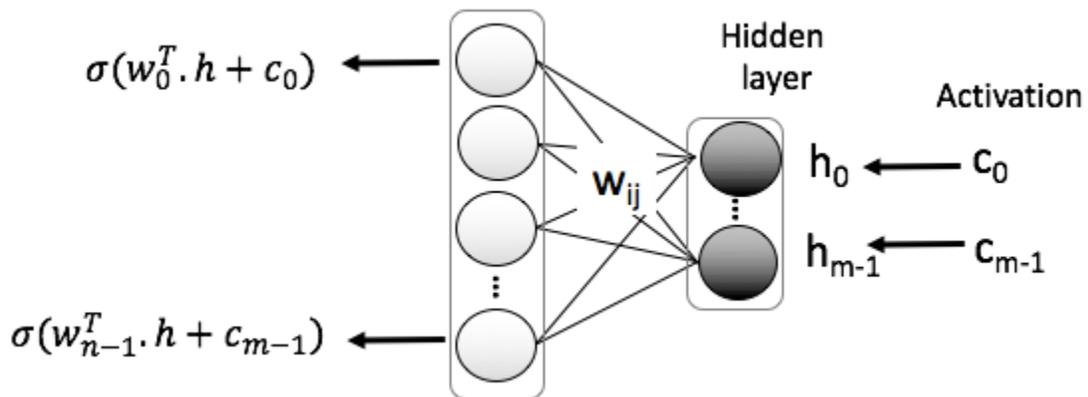
$$E(x) = -(x^T W x + b^T x) = -\sum_{i < j}^{n-1} w_{ij} x_i x_j - \sum_{i=0}^{n-1} b_i x_i$$

$$p(x=1) = \frac{1}{1 + e^{-\frac{E(x=1) - E(x=0)}{T}}}$$

$$p(v|h) = \prod_{j=0}^{n-1} p(v_j|x) \quad p(v_j=1|h) = \sigma\left(\sum_{i=0}^{n-1} w_{ij}h_i + b_j\right)$$



$$p(h|v) = \prod_{i=0}^{m-1} p(h_i|v) \quad p(h_i=1|v) = \sigma\left(\sum_{j=0}^{n-1} w_{ij}v_j + c_i\right)$$



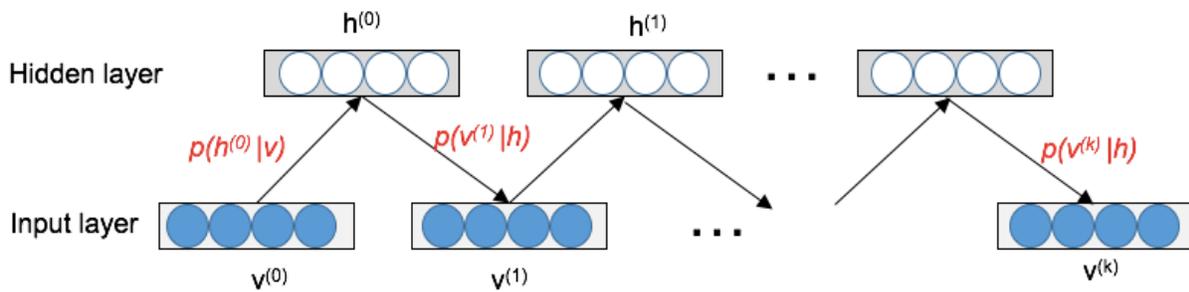
$$p(v|w) = \frac{1}{Z(w)} \tilde{p}(v|w)$$

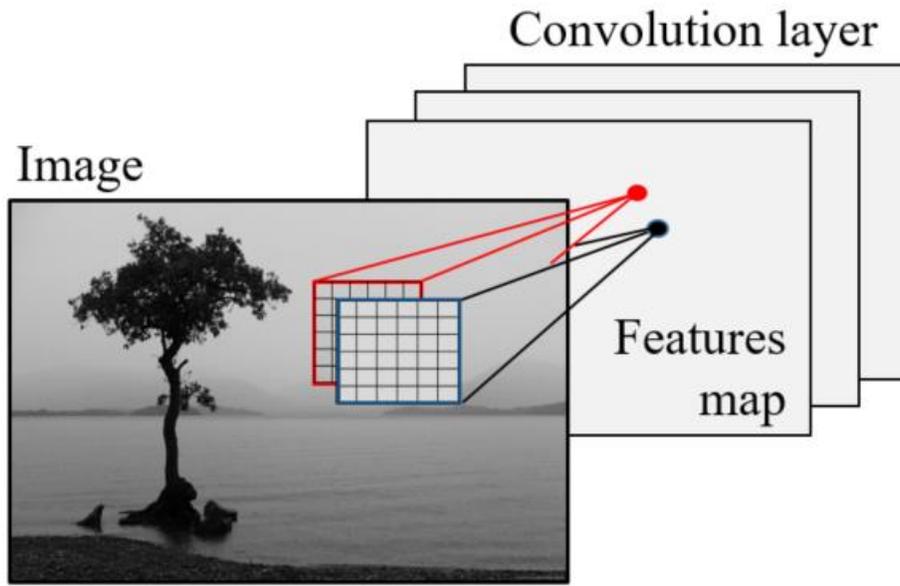
$$\frac{\partial \ln p(v|w)}{\partial w} = \frac{\partial \ln \tilde{p}(v|w)}{\partial w} - \frac{\partial Z(w)}{\partial w}$$

$$\frac{\partial \ln(\mathcal{L}(w|v))}{\partial w_{ij}} \triangleq \Delta_e = \frac{1}{n} (e_+ - e_-)$$

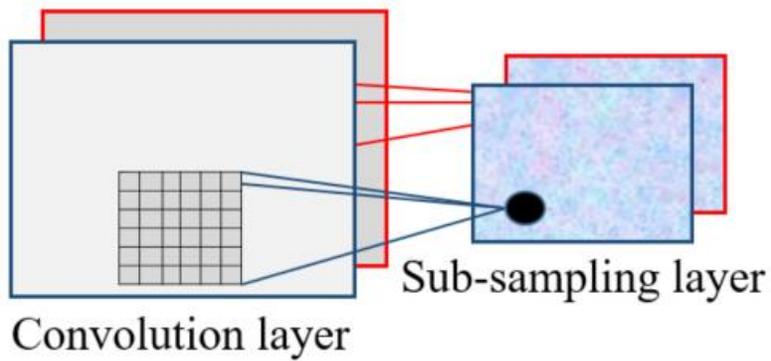
$$e_+ = v^{(0)} \cdot p(v^{(0)} | h)$$

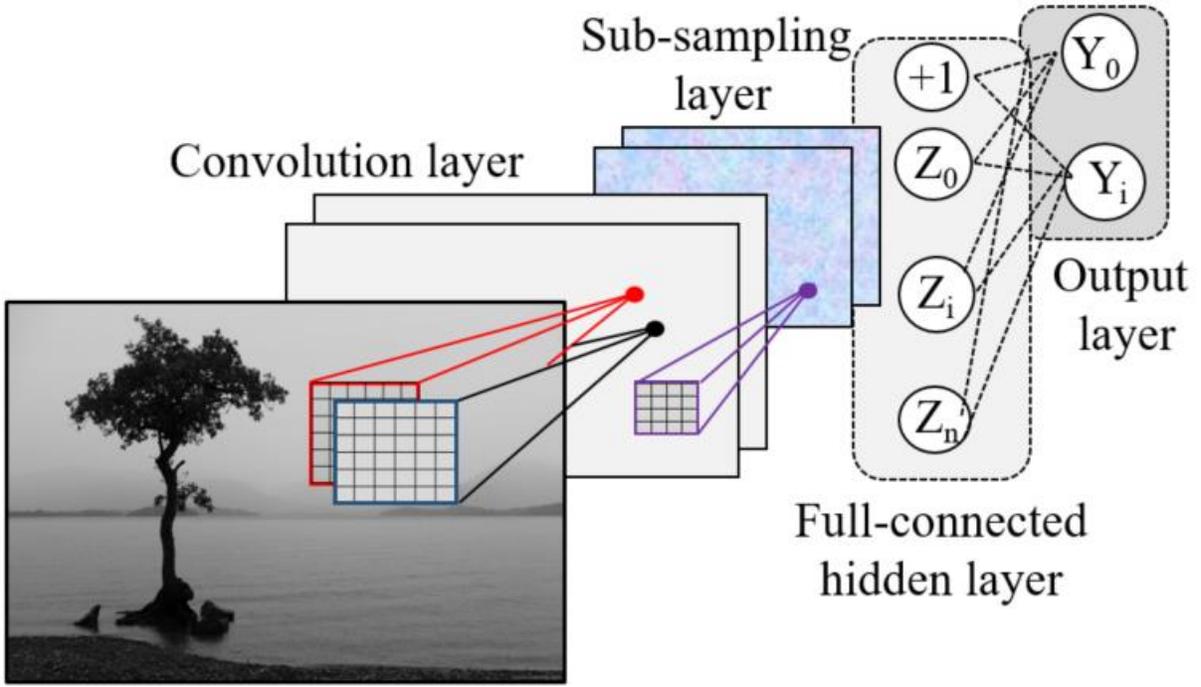
$$e_- = v^{(1)} \cdot p(v^{(1)} | h) \quad v^{(1)} = p(h^{(0)} | v) \quad h^{(0)} = p(v^{(0)} | h)$$



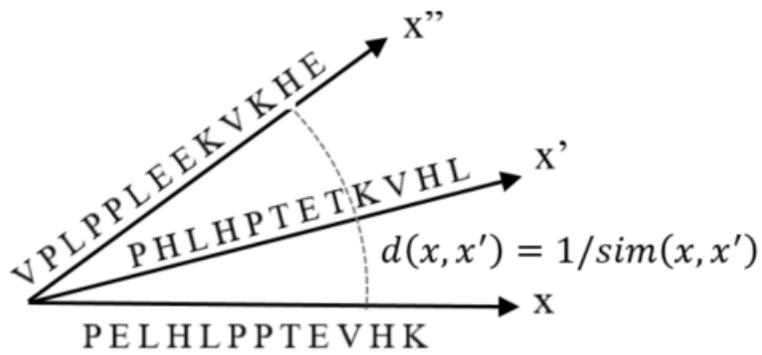
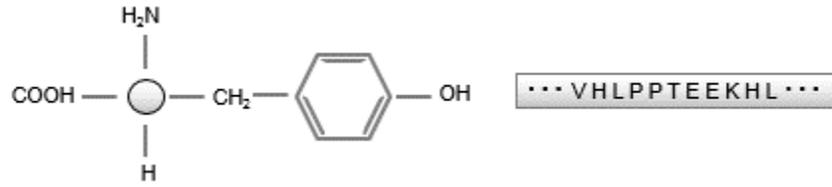


$$\tilde{z}_j = \sigma \left(w_0 + \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} w_{u,v} x_{j+u,i+v} \right)$$





Chapter 12: Kernel Models and SVM



$$\text{sim}(x_{cp}, x'_{c'p'}) = \frac{1}{mx} \sum_{i=1}^{mx} (c = c') \cap (p = p') \quad mx = \max(n, n')$$

$$f(x|w) = w_0 + \sum_{d=1}^D w_d \phi_d(x) \quad \phi_d : \mathbb{R} \rightarrow \mathbb{R}$$

$$K(x, x') = \phi(x) \cdot \phi(x') = \sum_{d=1}^D \phi_d(x) \phi_d(x')$$

$$K(x, x') = x^T x' = \sum_{d=1}^D x_d \cdot x'_d$$

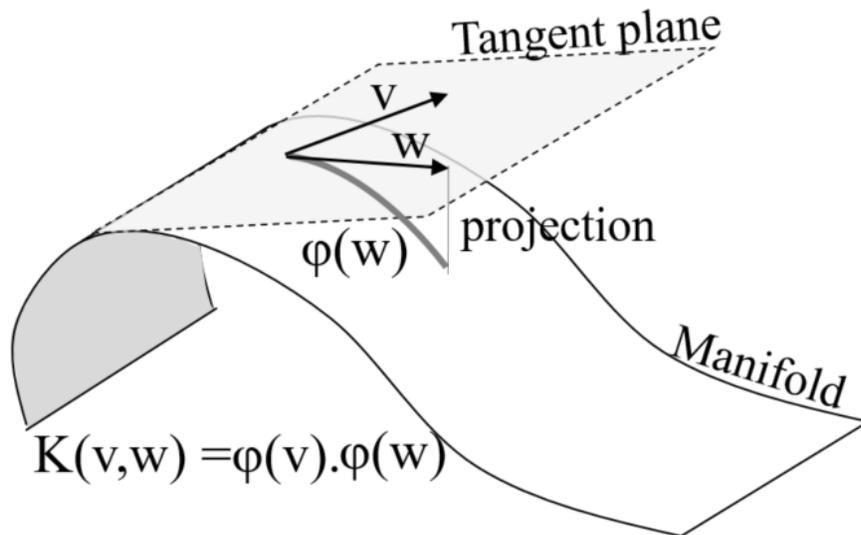
$$K(x, x') = (\gamma x^T x' + c)^n \quad \gamma > 0, c \geq 0$$

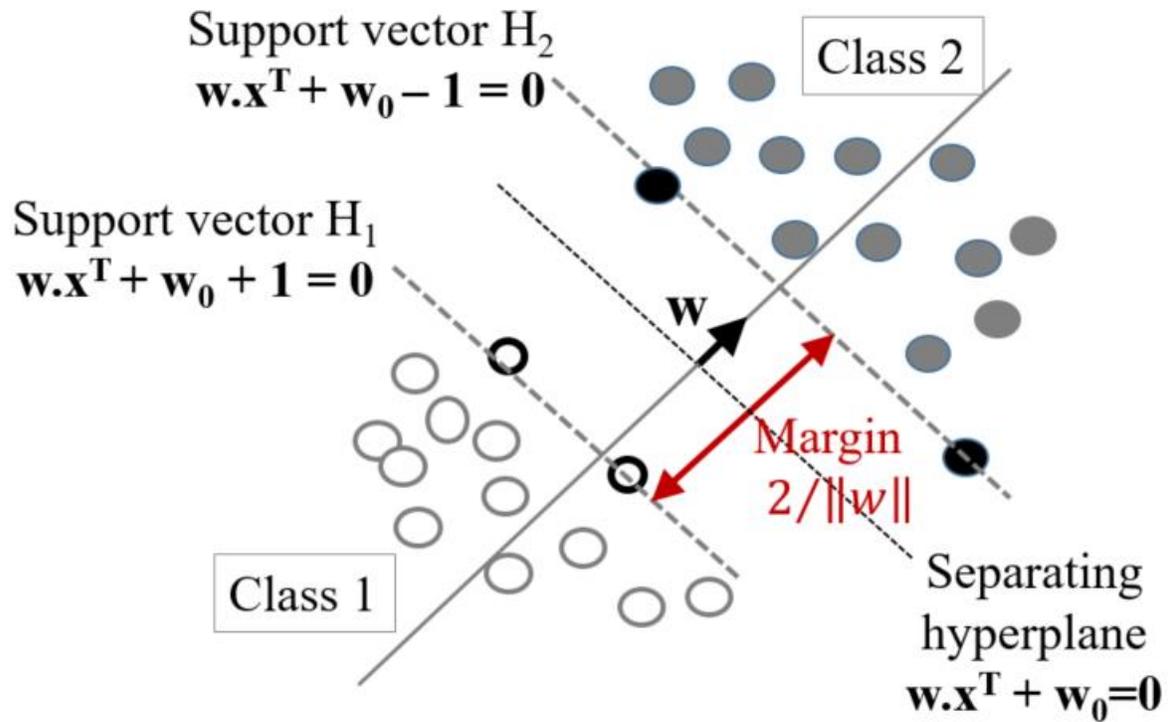
$$K(x, x') = \tanh(\gamma x^T x' + c) \quad \gamma > 0, c \geq 0$$

$$K(x, x') = e^{-\gamma \|x - x'\|^2} \quad \gamma > 0$$

$$K(x, x') = e^{-\gamma \|x - x'\|^n} \quad \gamma > 0$$

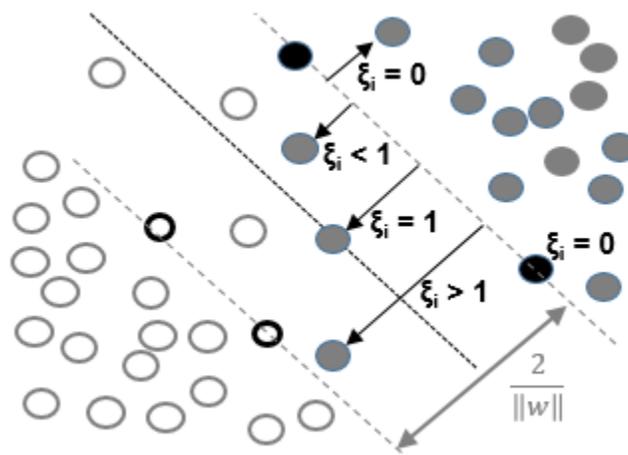
$$K(x, x') = -\log(1 + \|x - x'\|^n)$$





$$y_i (w^T x + w_0) \geq 1 \quad \forall i$$

$$\min_{w, w_0} \left\{ \frac{w^T w}{2} \right\} \text{ subject to } y_i (w^T x + w_0) \geq 1 \quad \forall i$$



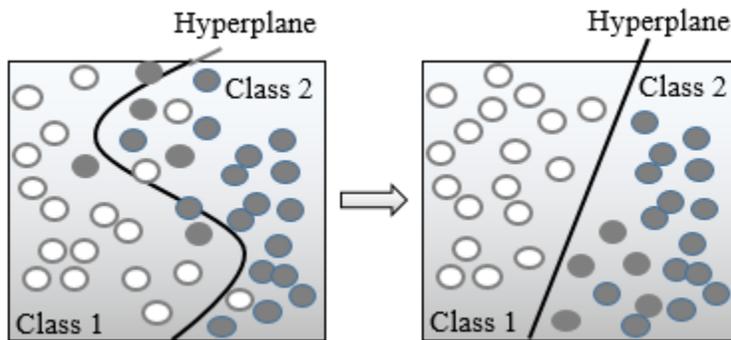
$$\min_{w, \xi} \left\{ \frac{w^T w}{2} + C \sum_{i=0}^{n-1} \xi_i \right\}$$

$$\xi_i \geq 0, y_i (w^T x + w_0) \geq 1 - \xi_i \quad \forall i$$

$$\min_{w, w_0} \left\{ \frac{w^T w}{2} + C \sum_{i=0}^{n-1} \mathcal{L}_i \right\} \mathcal{L}_i = |1 - y_i (w^T x + w_0)|$$

$$\min_{w, \rho, \xi} \left\{ \frac{w^T w}{2} - \rho + \frac{1}{vn} \sum_{i=0}^{n-1} \xi_i \right\}$$

$$\xi_i \geq 0, y_i (w^T x + w_0) \geq \rho - \xi_i \quad \forall i$$



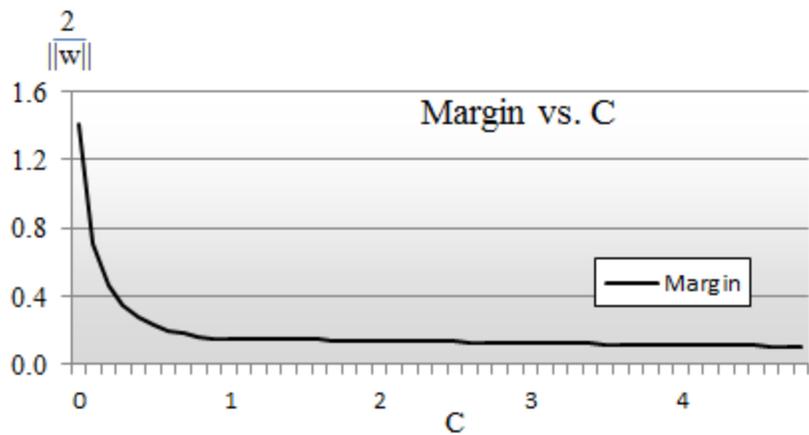
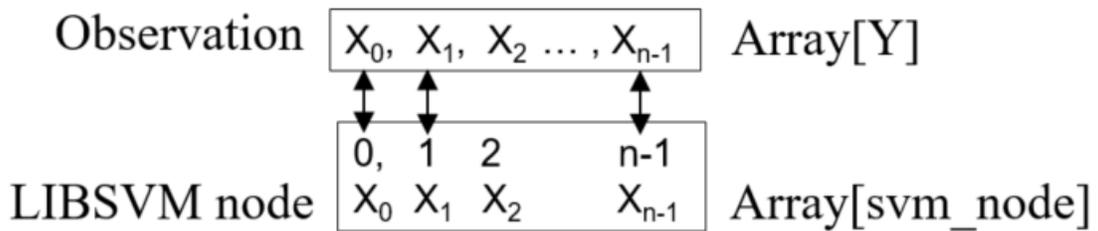
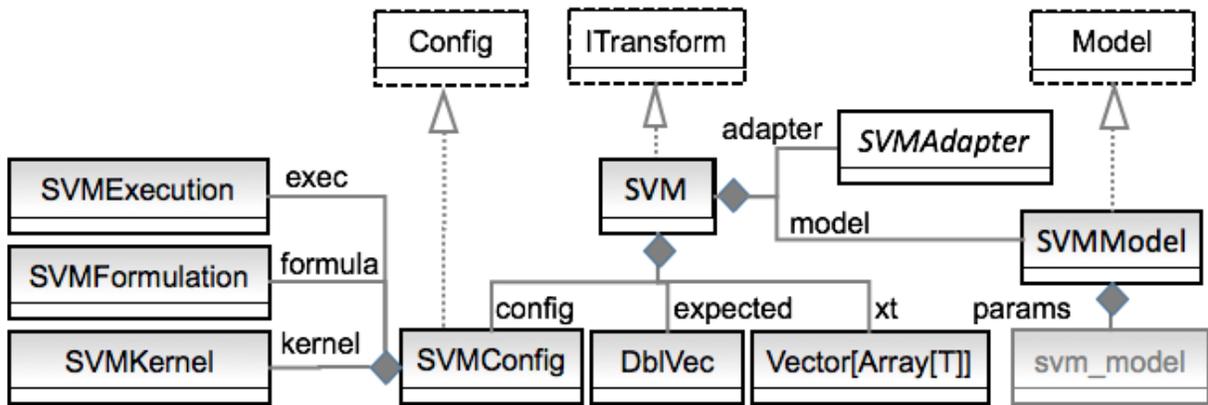
$$y_i (w^T \phi(x) + w_0) \geq 1 - \xi_i \quad \xi_i \geq 0 \quad \forall i$$

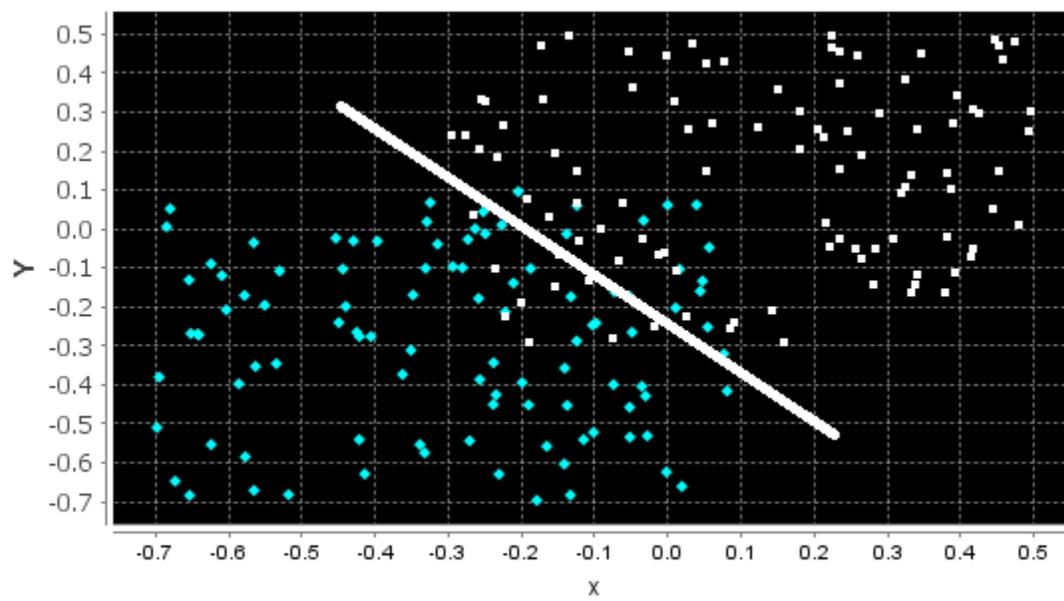
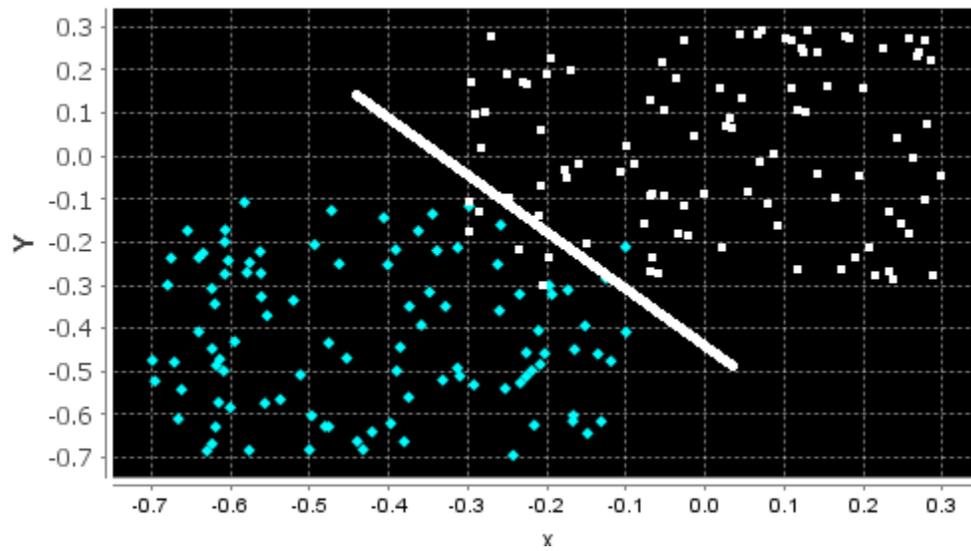
$$w^* = \sum_{i=0}^{n-1} \alpha_i y_i \phi(x_i)$$

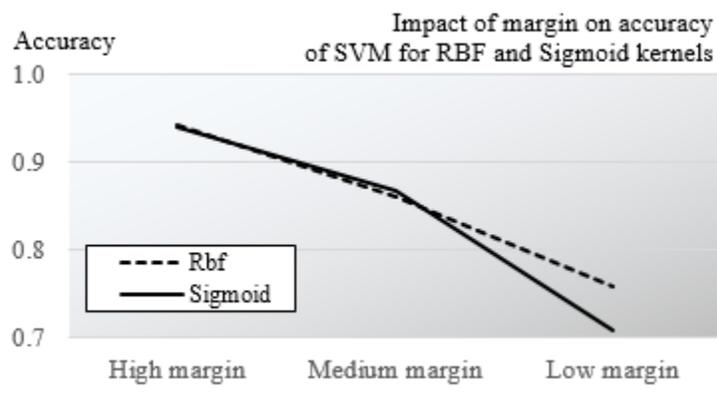
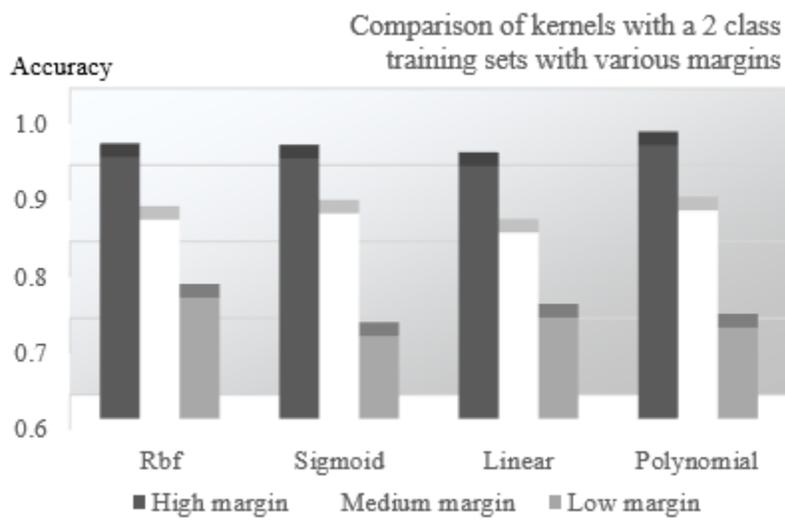
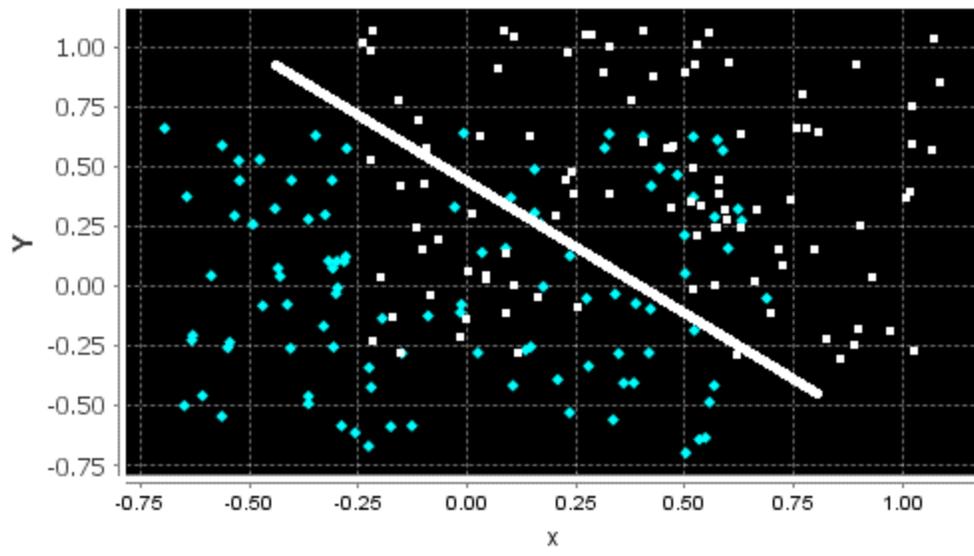
$$y_i (w^T \phi(x) + w_0) = y_i \left(\sum_{i=0}^{n-1} \alpha_i y_i K(x_i, x) + w_0 \right) \geq 1$$

$$K(x_i, x) = \phi(x_i) \phi(x) \quad \forall i$$

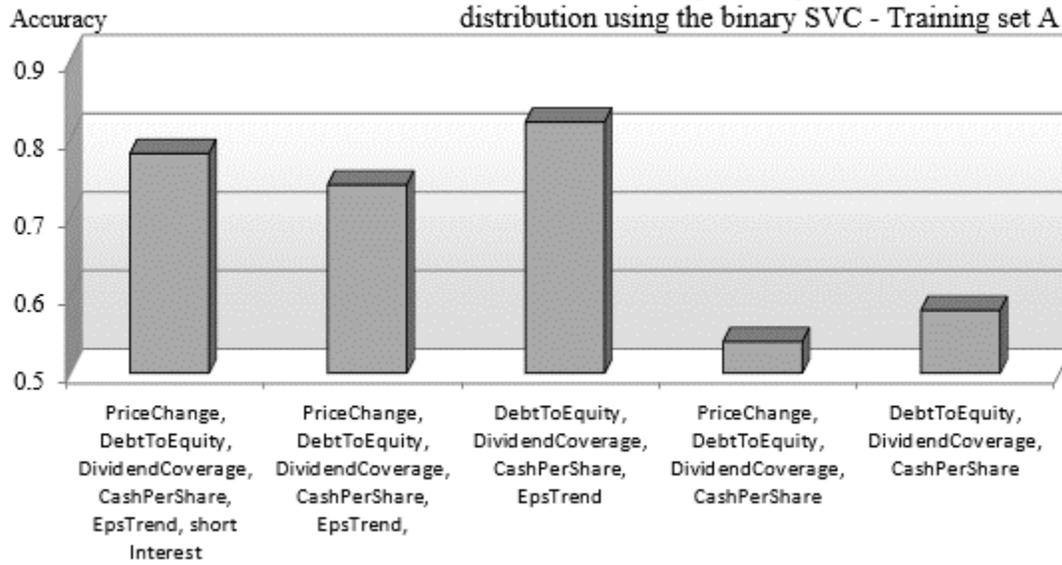
$$\begin{aligned}
 K(x_i, x) &= (1 + x^T x')^2 \\
 &= 1 + 2x_1 x'_1 + 2x_2 x'_2 + 2x_1 x'_1 x_2 x'_2 + (x_1 x'_1)^2 + (x_2 x'_2)^2 \\
 &= \phi_1(x) \cdot \phi_1(x') + \phi_2(x) \cdot \phi_2(x') + \phi_3(x) \cdot \phi_3(x') + \dots \\
 \phi_2(x) &= 1, \phi_2(x) = \sqrt{2}x_1, \phi_3(x) = \sqrt{2}x_2, \phi_4(x) = x_1^2 \dots
 \end{aligned}$$



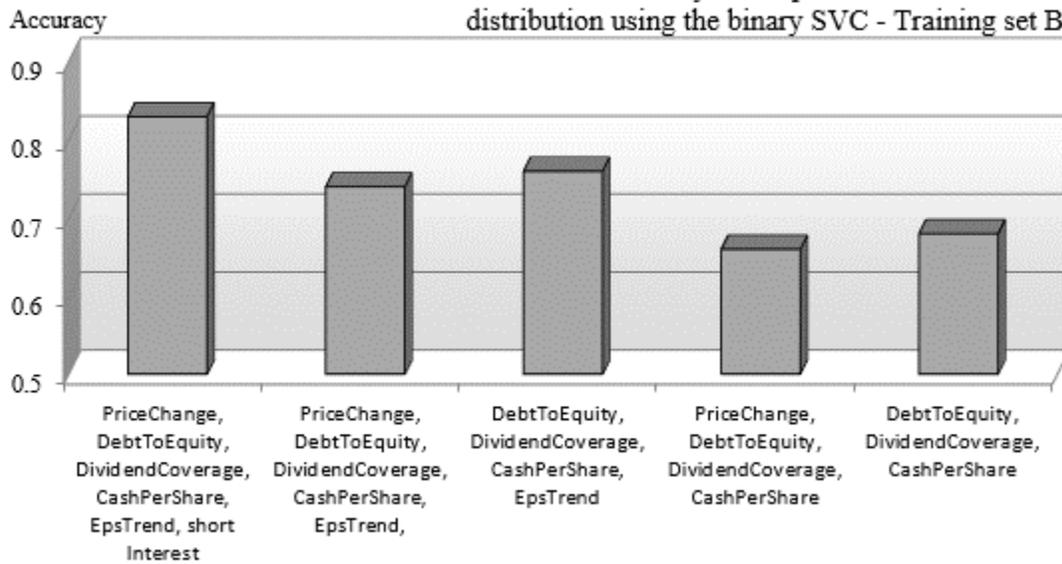


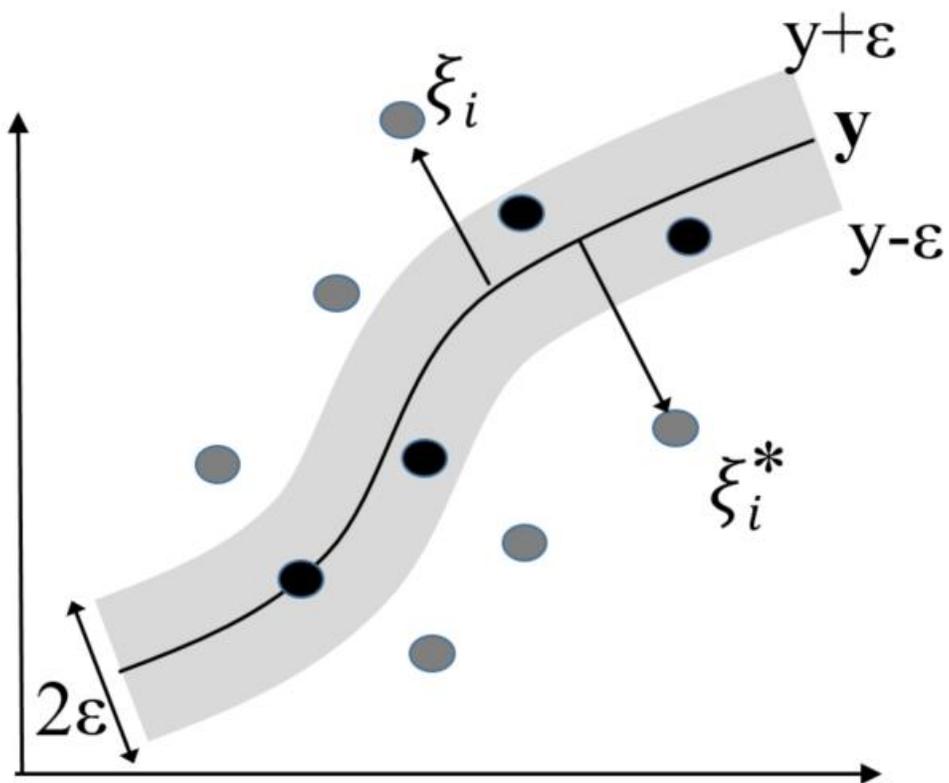
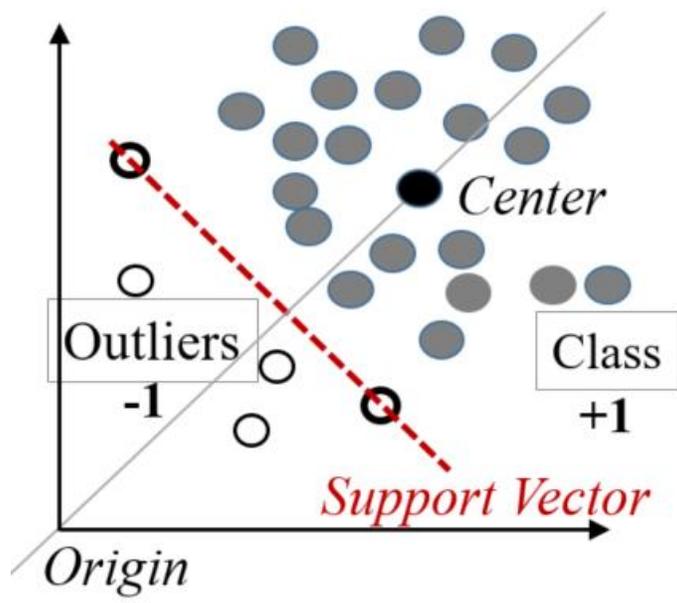


Accuracy of the prediction of dividend distribution using the binary SVC - Training set A



Accuracy of the prediction of dividend distribution using the binary SVC - Training set B

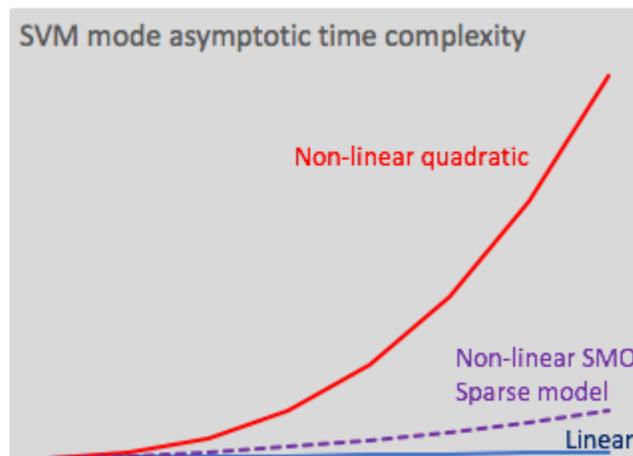
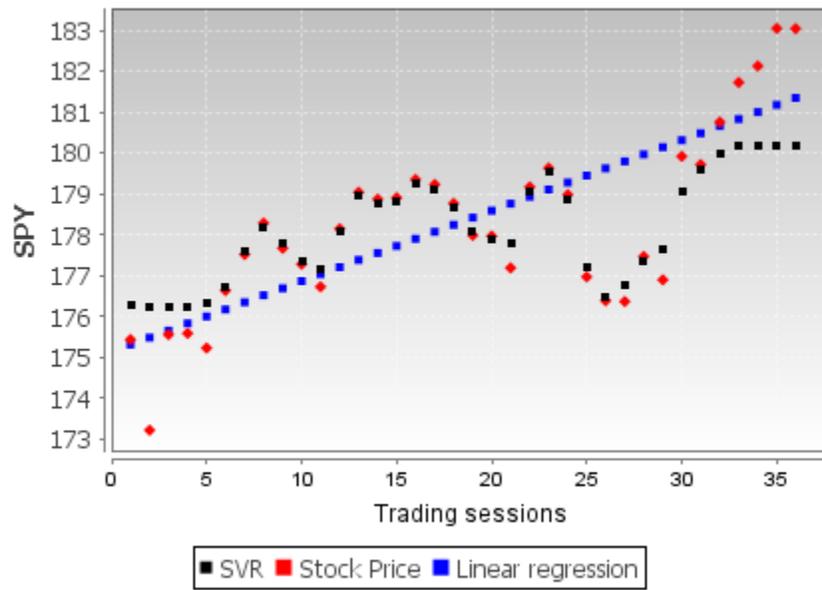




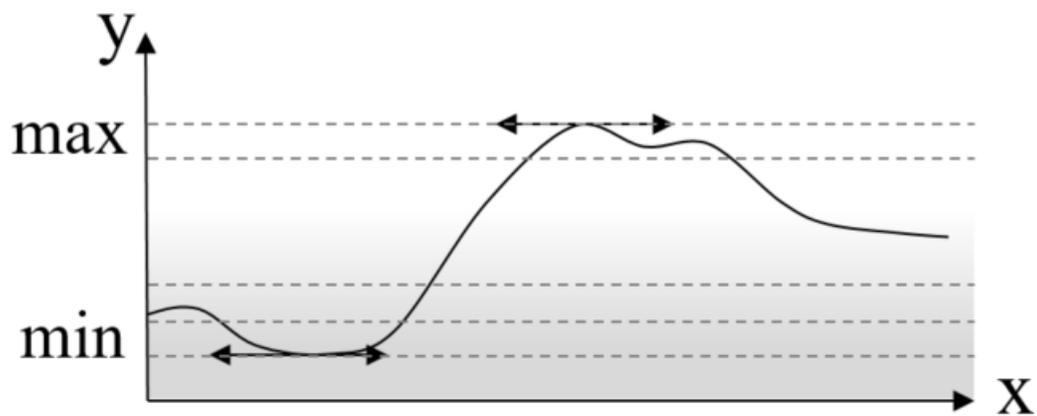
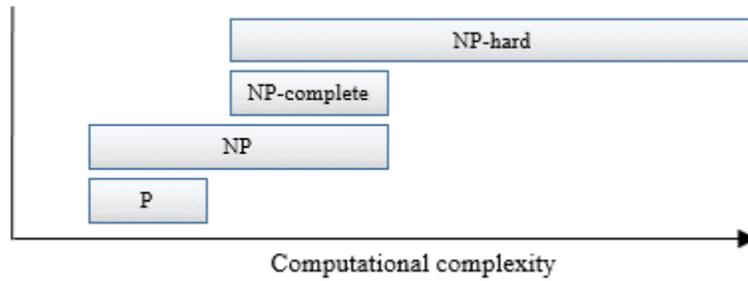
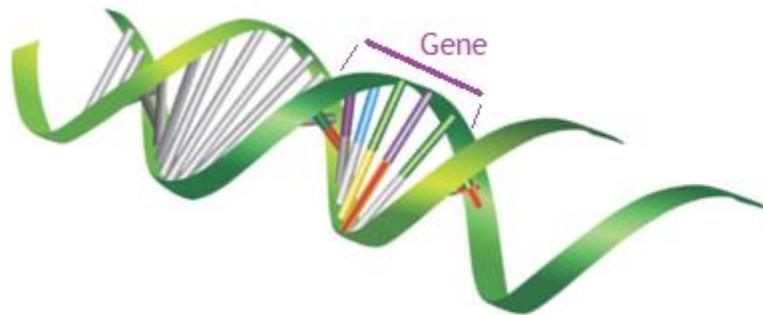
$$\min_{w, \xi, \xi^*} \left\{ \frac{w^T w}{2} + C \sum_{i=0}^{n-1} (\xi_i + \xi_i^*) \right\}$$

$$- \epsilon - \xi_i^* \leq w^T \phi(x_i) + w_0 - y_i \leq \epsilon + \xi_i \quad \forall i$$

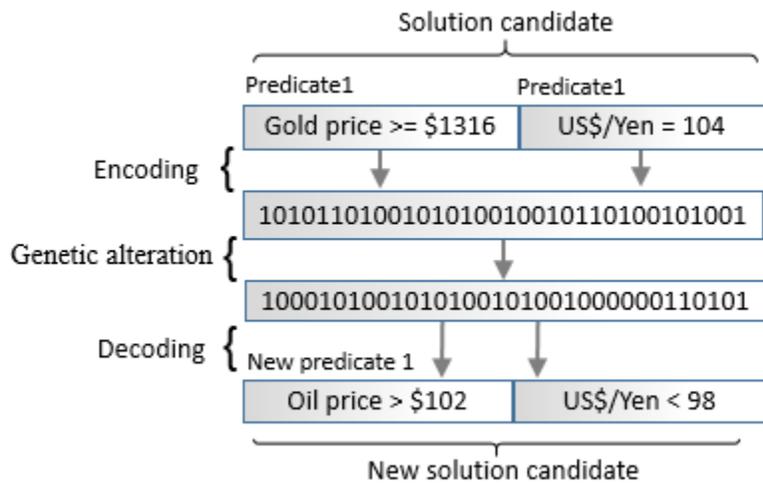
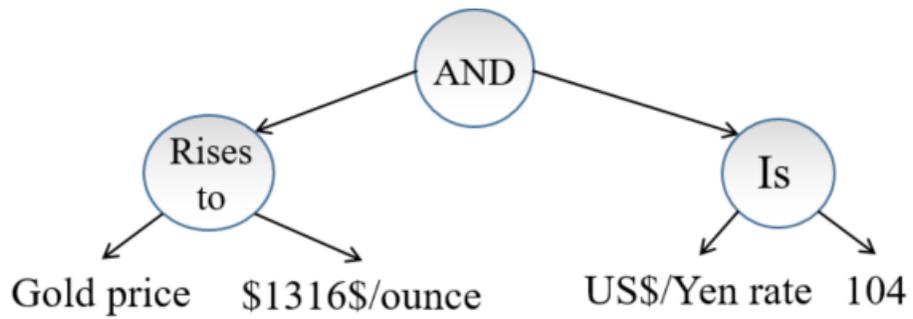
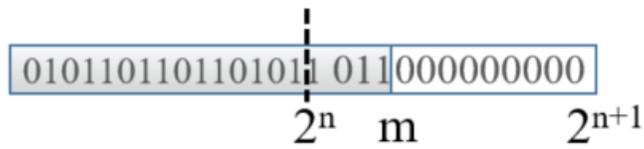
$$\hat{y}(x) = \sum_{i=0}^{n-1} \alpha_i K(x_i, x) + \hat{w}_0$$

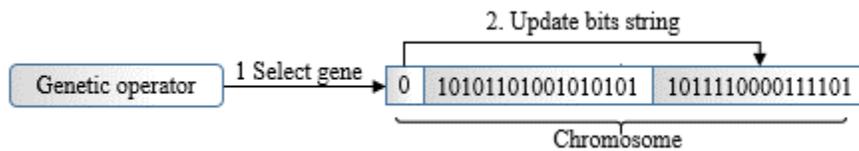
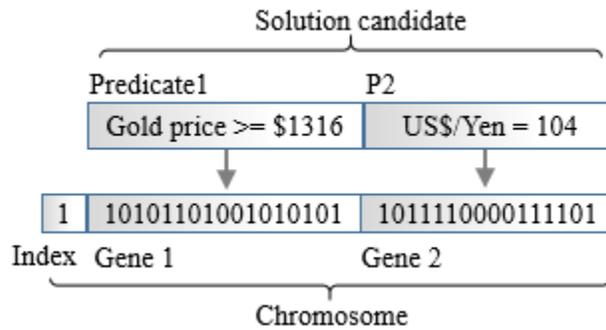
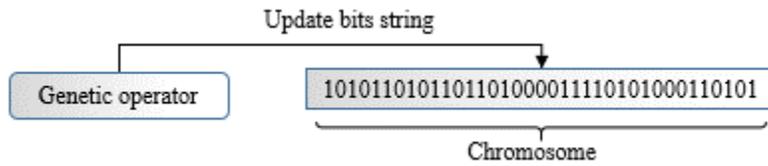
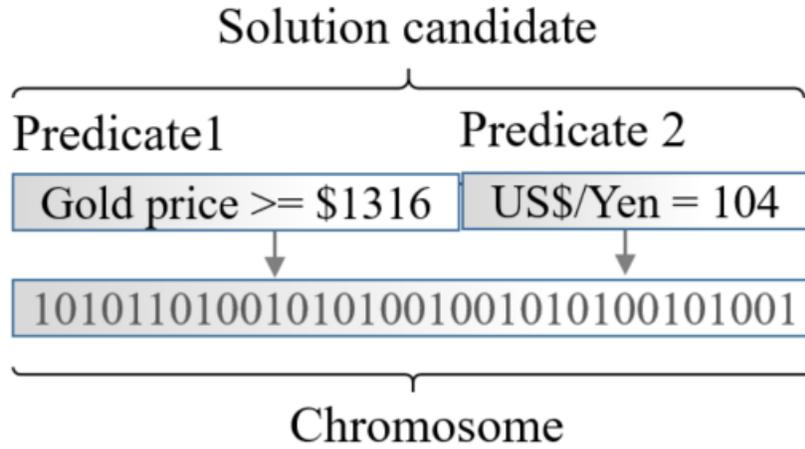


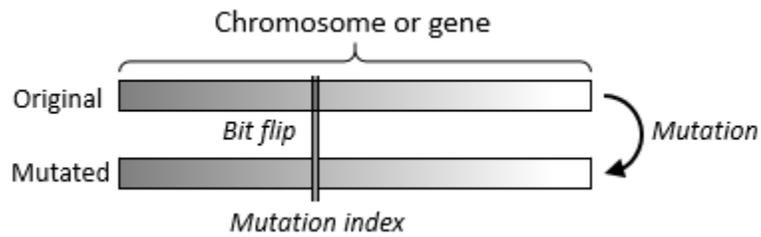
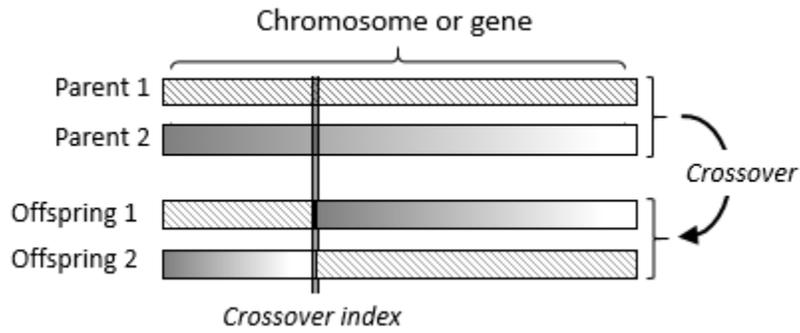
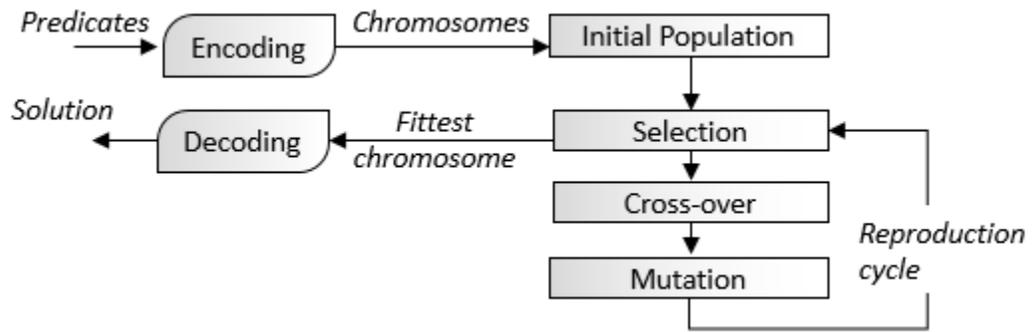
Chapter 13: Evolutionary Computing

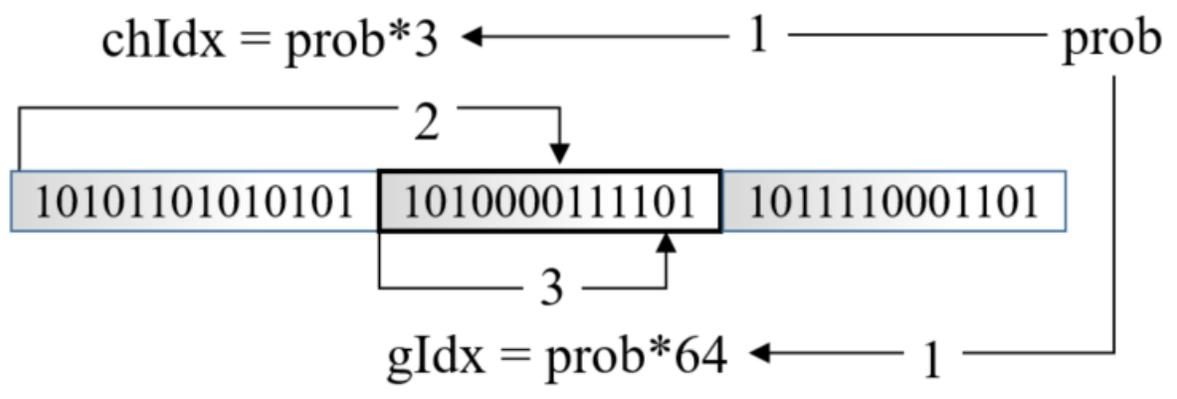
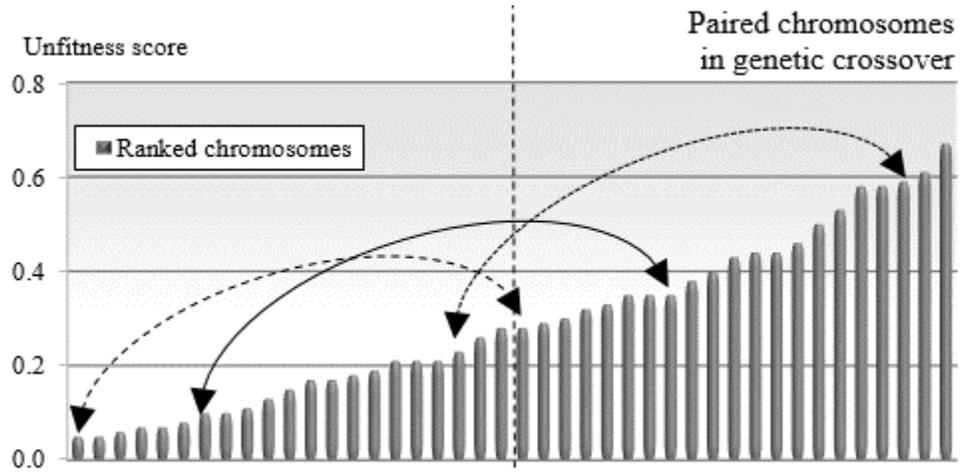
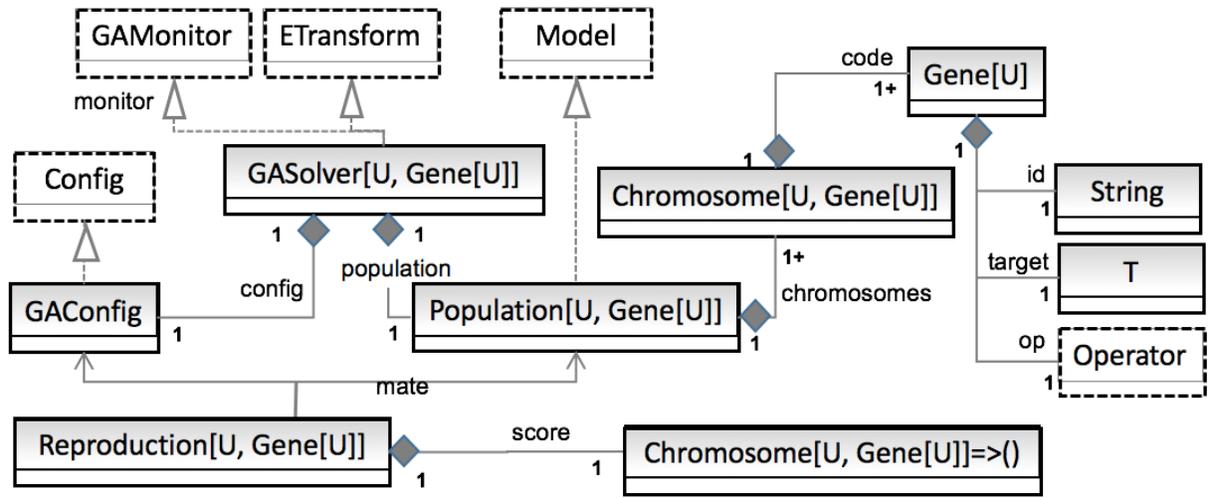


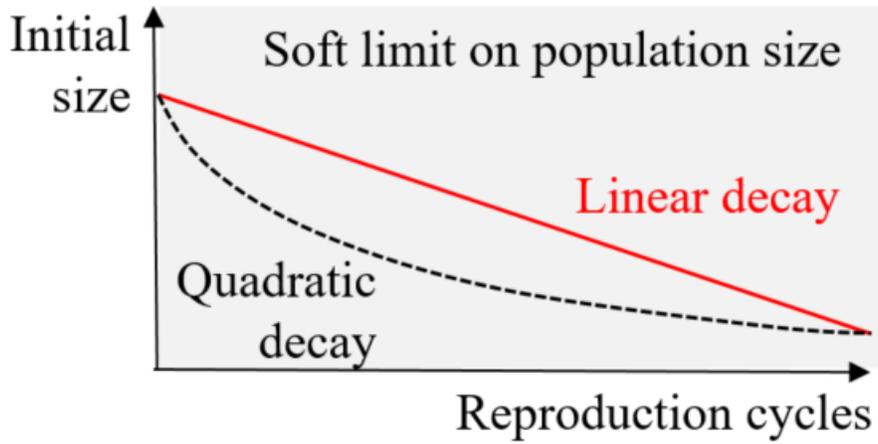
$$step = \frac{max - min}{2^n}$$



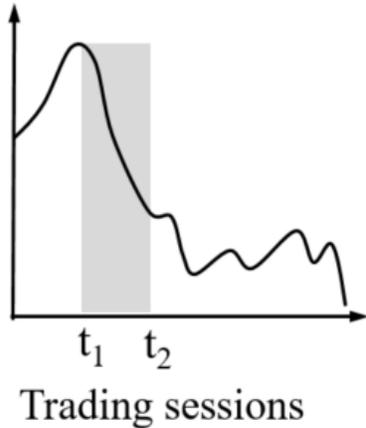




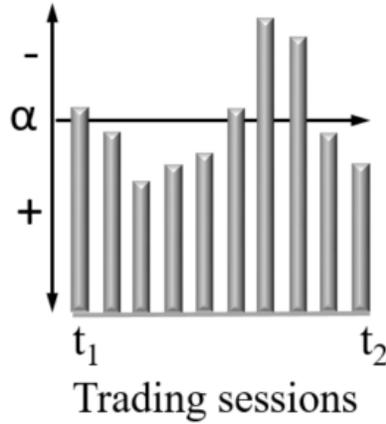




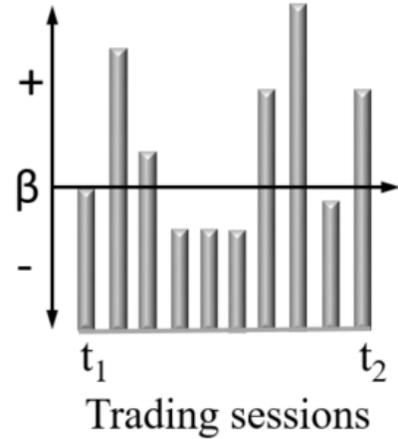
Price of security



Rel. volume v^m



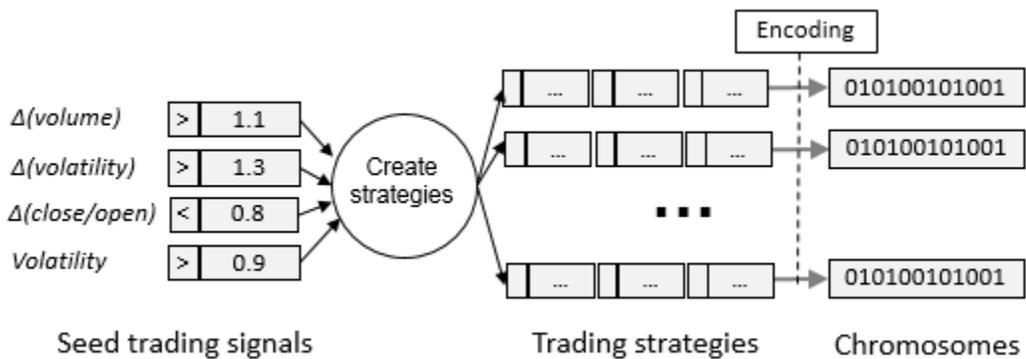
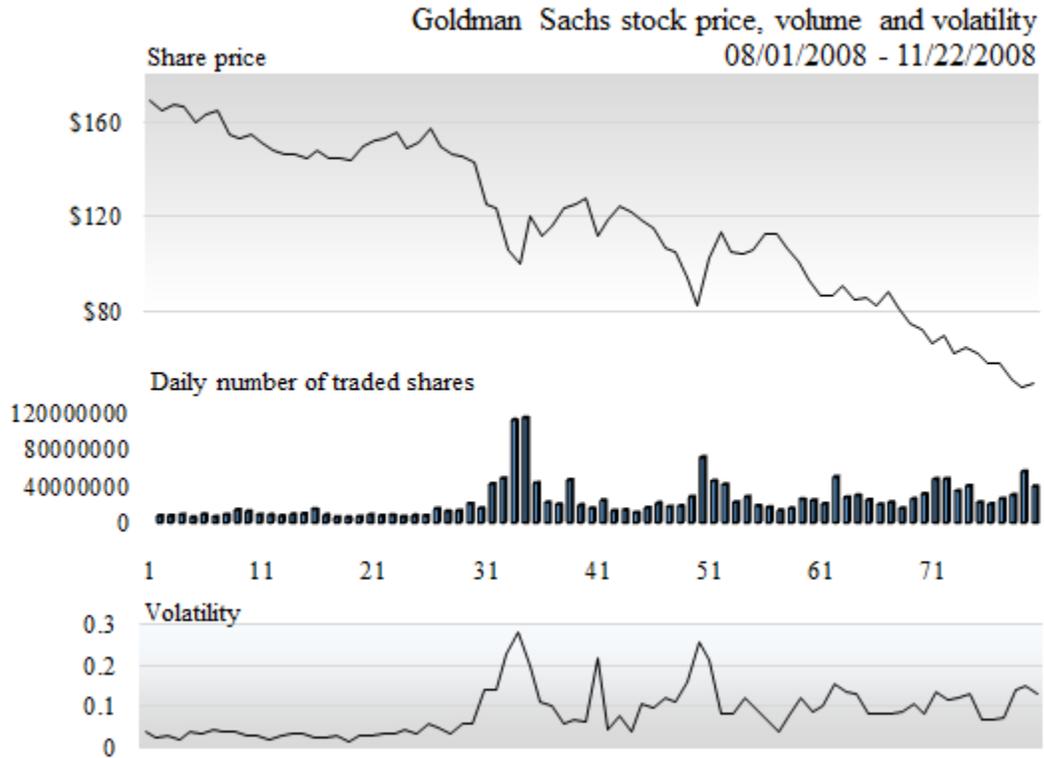
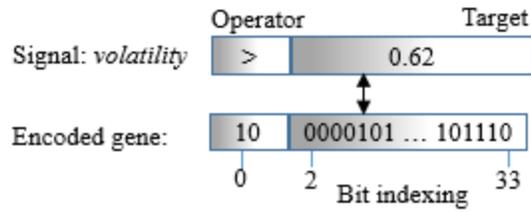
Rel. volatility v^l

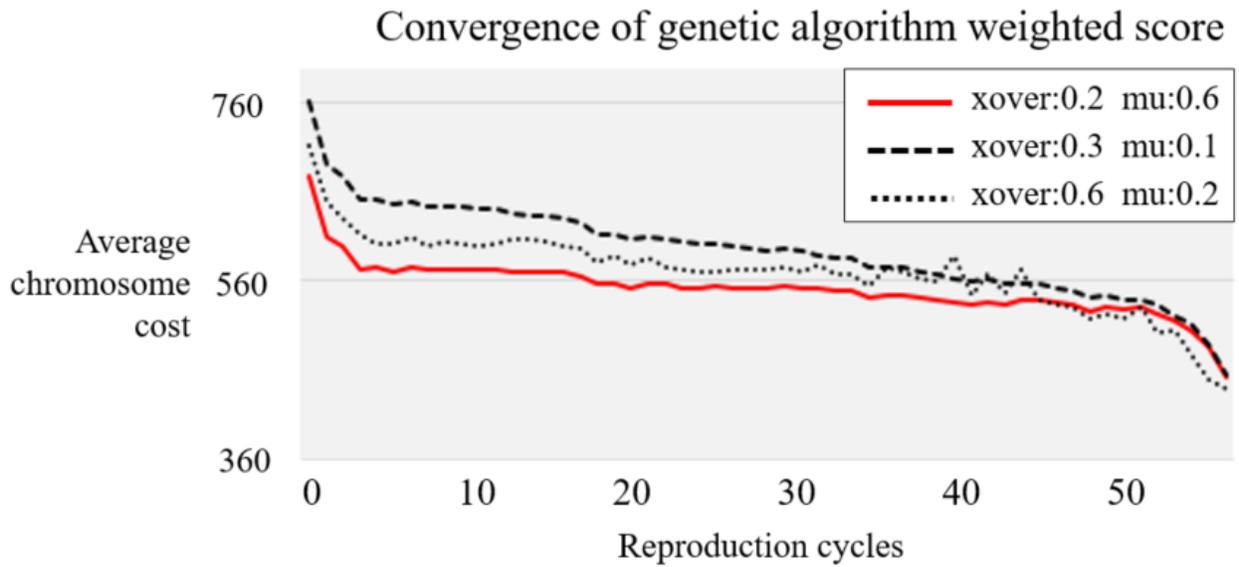
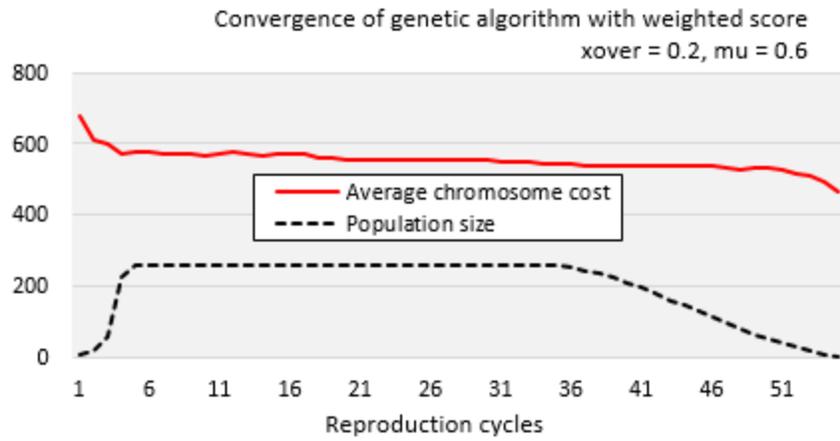


$$w_t = -\Delta p_t$$

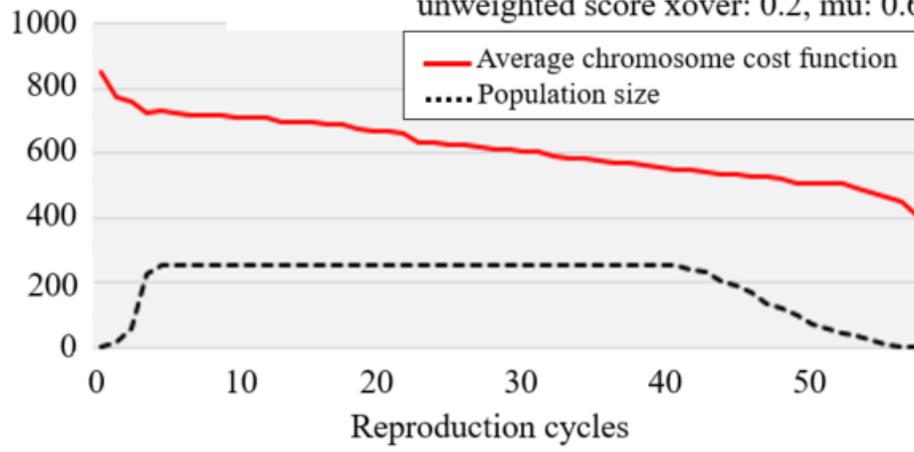
$$C(p, v^m, v^l | \alpha, \beta) = \sum_{t=0}^{n-1} (\alpha - v_t^m) w_t + (v_t^l - \beta) w_t$$



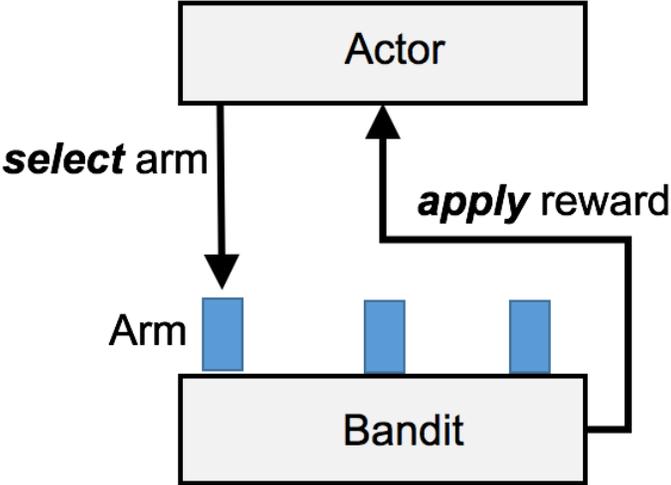
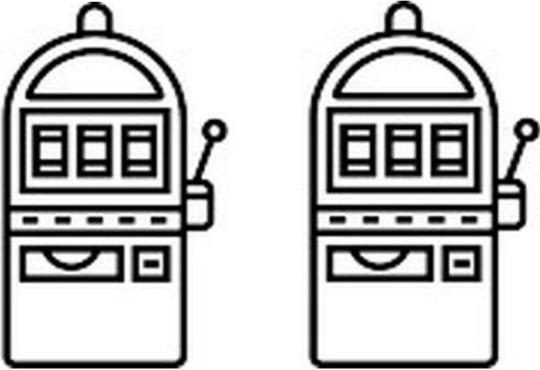




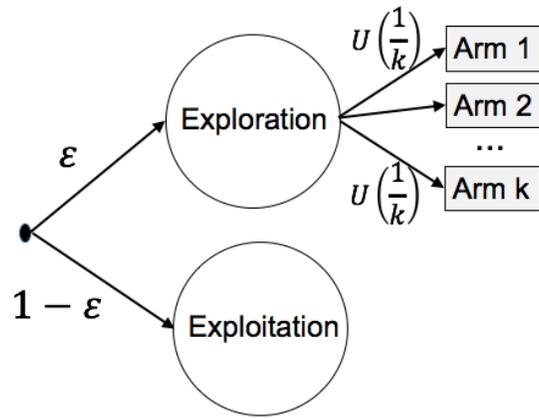
Convergence of genetic algorithm
unweighted score xover: 0.2, mu: 0.6



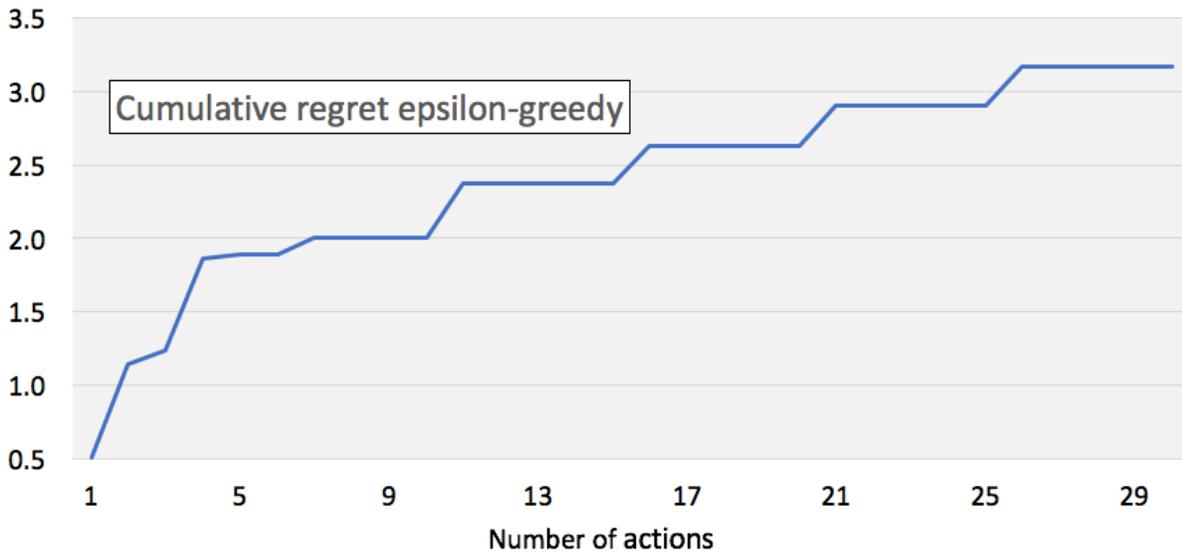
Chapter 14: Multiarmed Bandits



$$E[R] = T\mu^* - \sum_{t=0}^{T-1} \mu_{a_t}$$

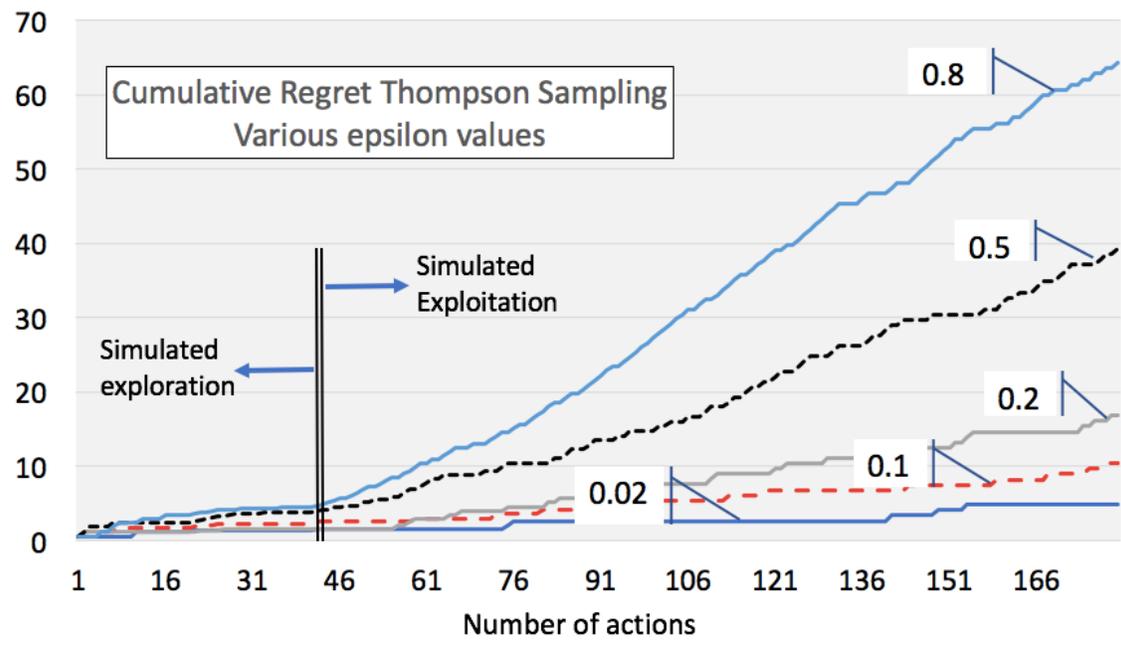
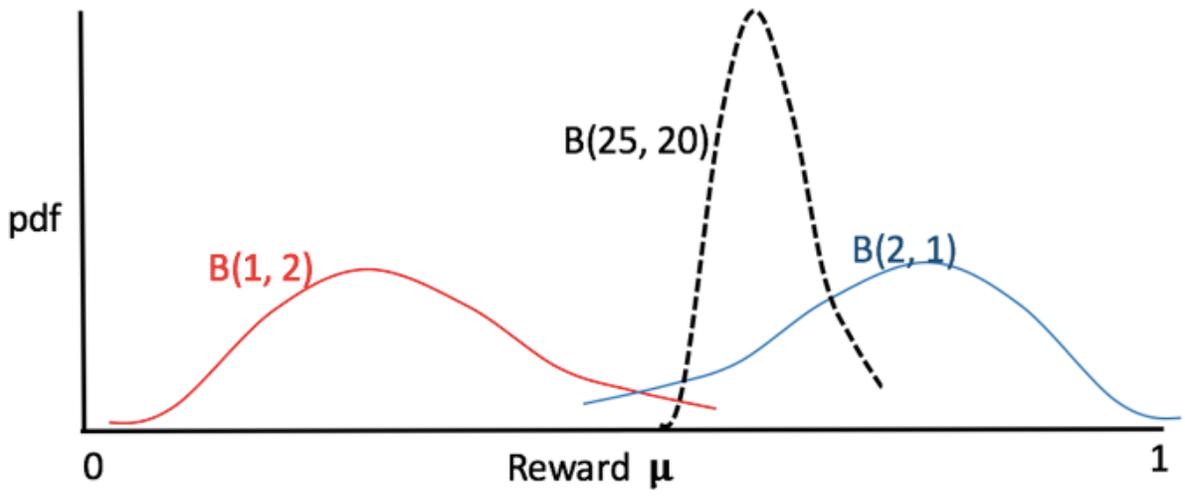


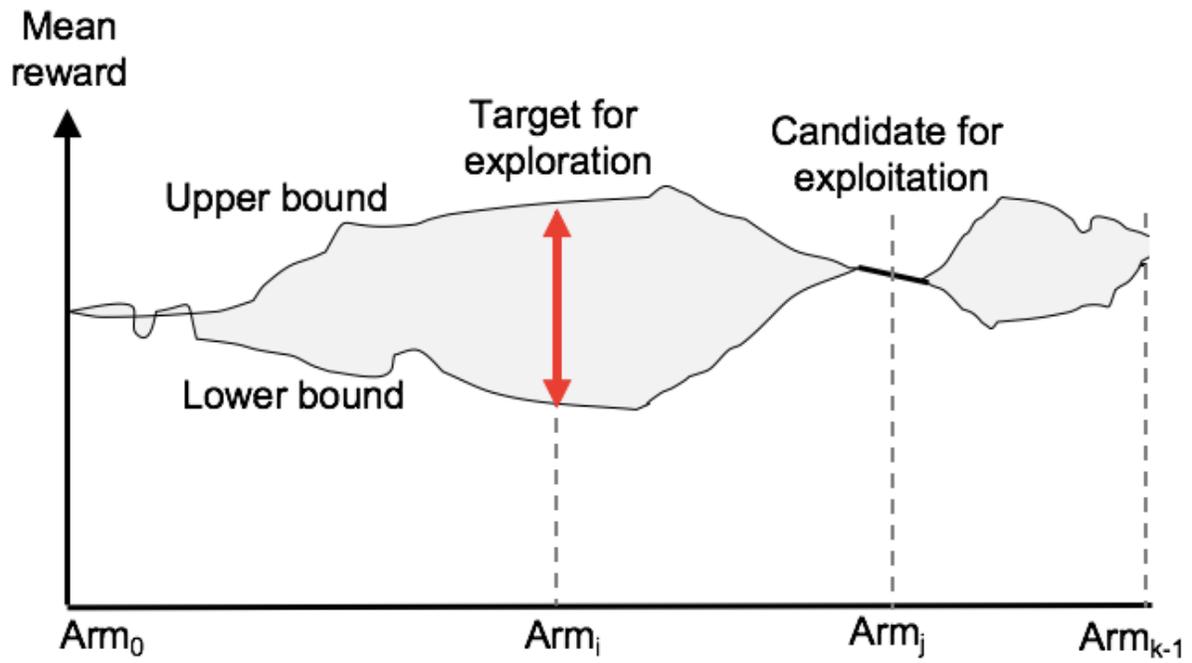
$$E[R] = T\varepsilon \left(\mu^* - \frac{1}{K} \right) \sum_{j=0}^{K-1} \mu_j$$



$$m = \arg \max_{0 \leq j < k} f(\cdot | w_j)$$

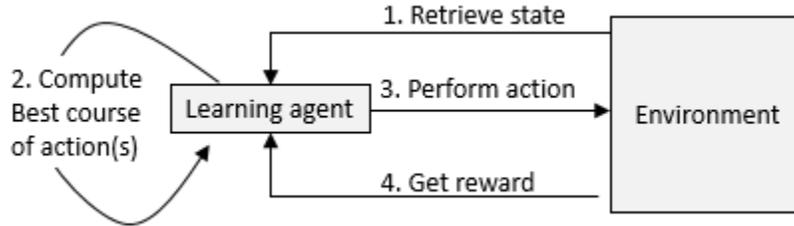
$$f(x | w) = \frac{1}{1 + e^{-\sum x_i w_j}}$$





$$l = \arg \max_{0 \leq j < k} f \left\{ \mu_j + \sqrt{2 \frac{\log t}{n_j}} \right\}$$

Chapter 15: Reinforcement Learning

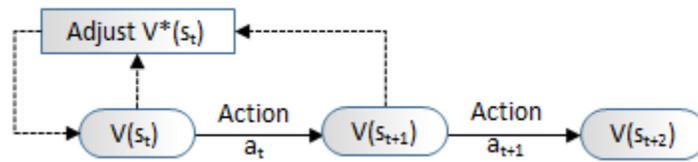


$$R_t = \sum_{k=0}^{+\infty} \gamma^k r_{t+1+k}$$

$$V^\pi(s_t) = E\{R_t | s_t\}$$

$$V^\pi(s_t) = \sum_{a \in A} \pi_t \sum_k \{p_k (r_k + \gamma \cdot V^\pi(s_k))\}$$

$$V^*(s_t) = \max_{\pi} V^\pi(s_t)$$

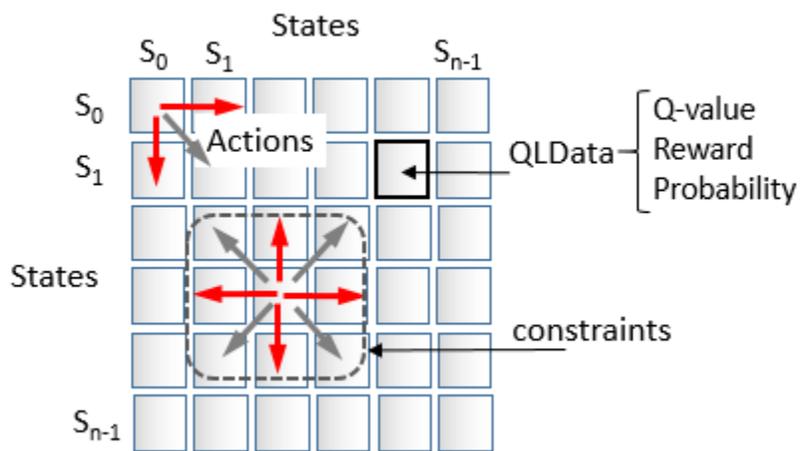
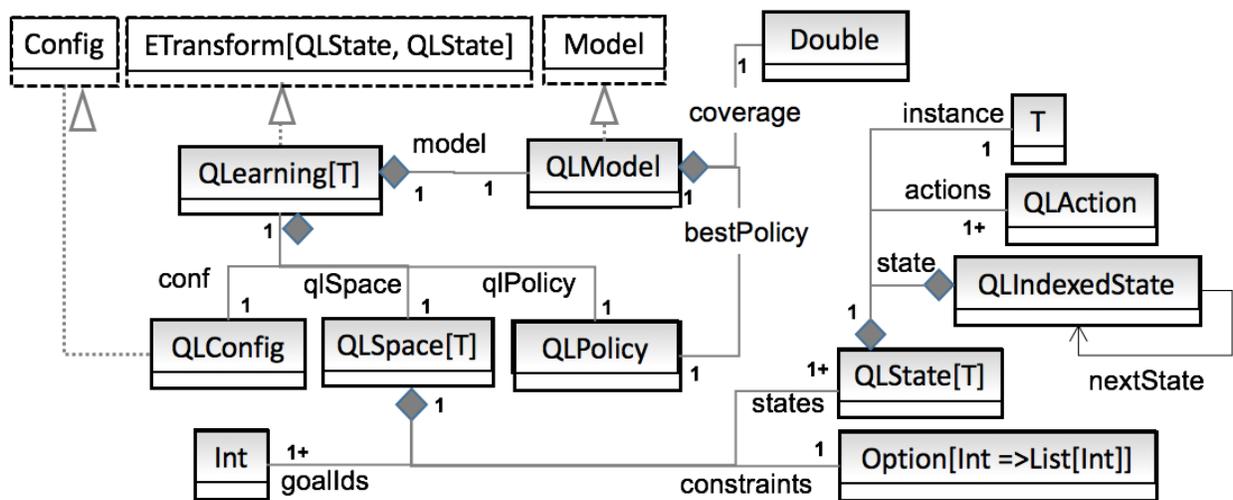


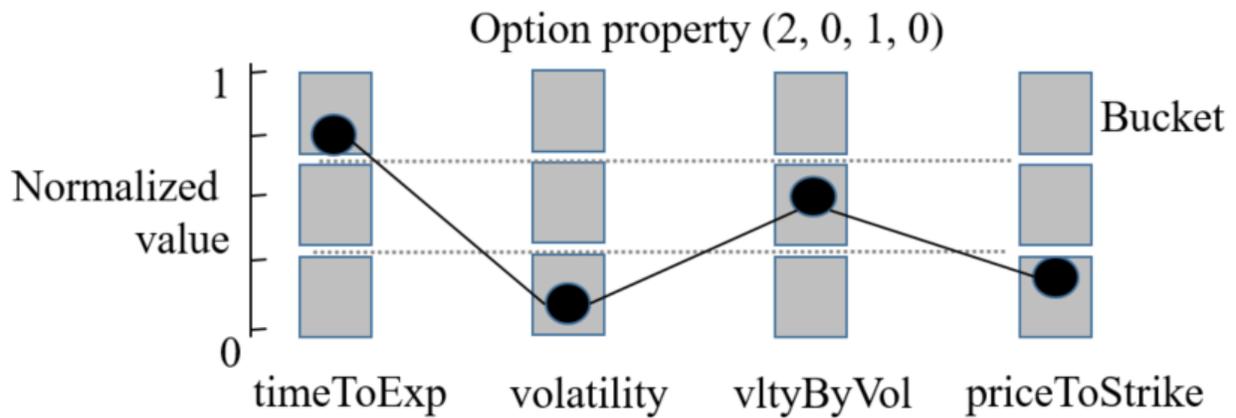
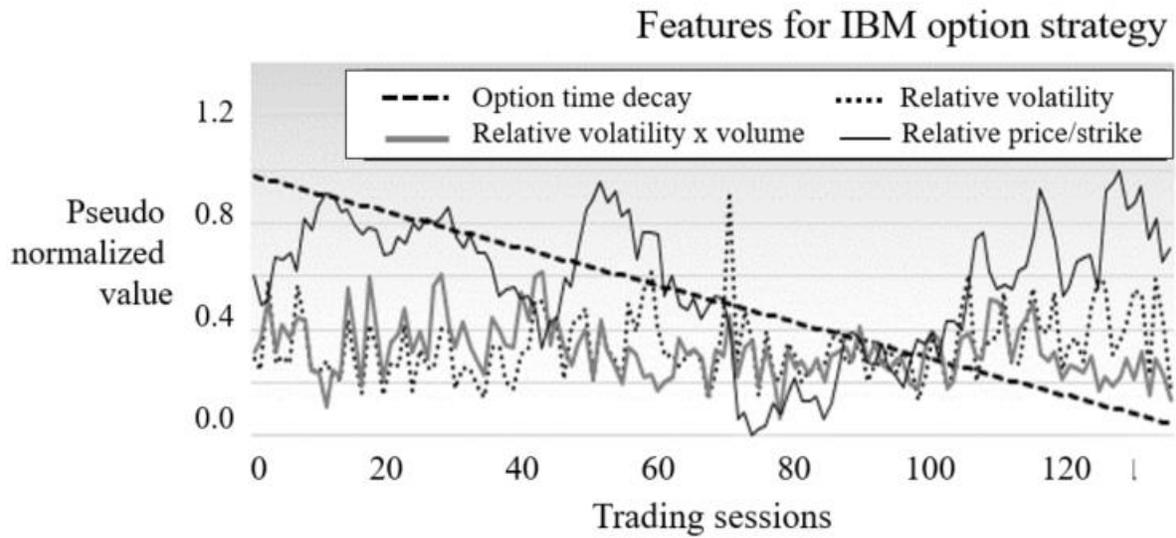
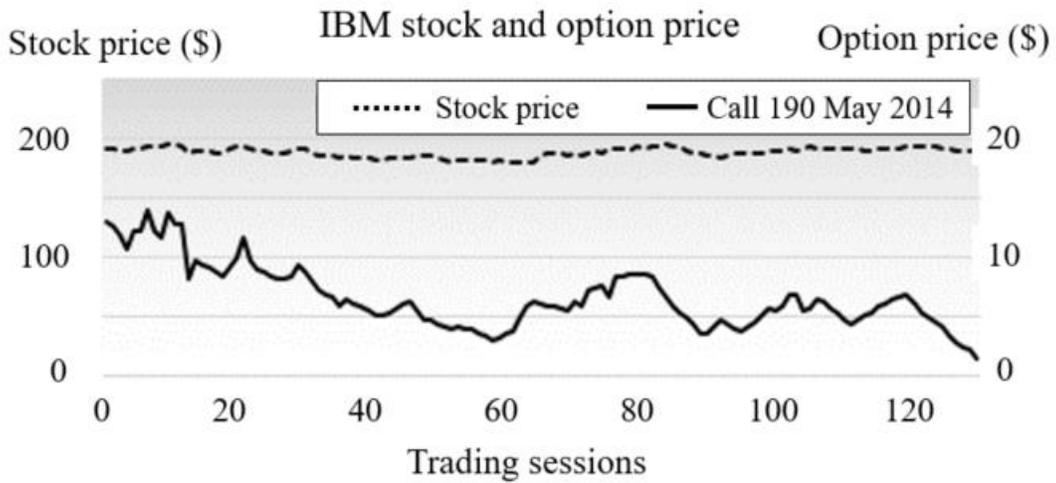
$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

$$\hat{V}^\pi(s_t) = V^\pi(s_t) + \alpha \delta_t$$

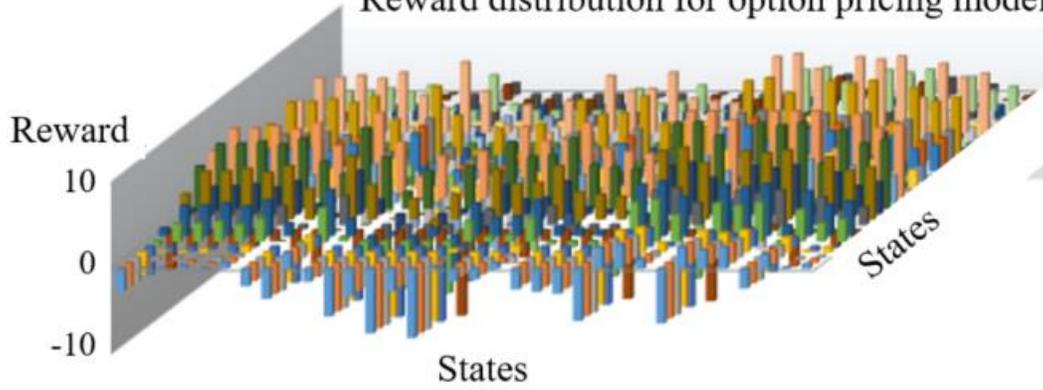
$$Q_t^\pi = Q^\pi(s_t, a_t) = E(R_t | s_t, a_t)$$

$$\tilde{Q}_t^\pi = Q_t^\pi + \alpha \left[r_{t+1} + \gamma \max_{a_{t+1}} Q_{t+1}^\pi - Q_t^\pi \right] Q_t^\pi = Q^\pi(s_t, a_t)$$

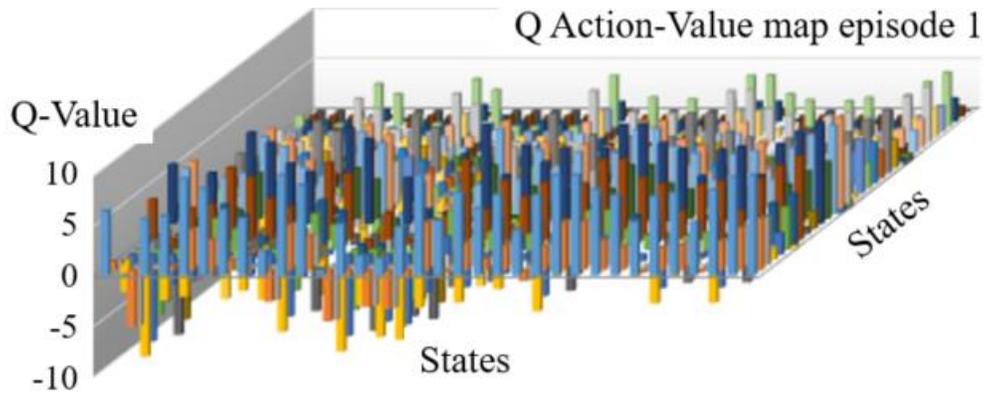




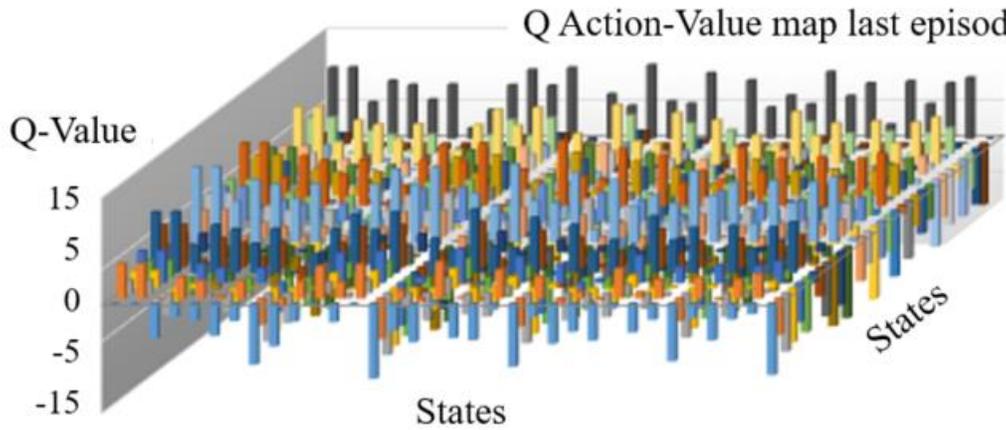
Reward distribution for option pricing model

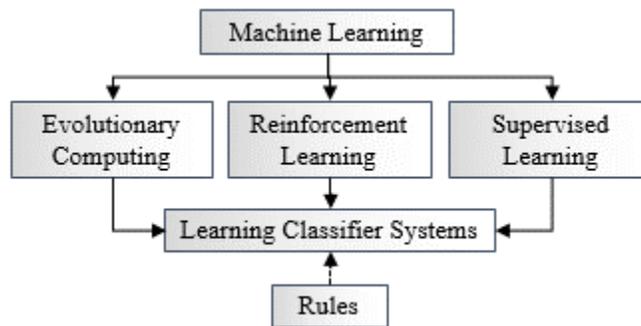
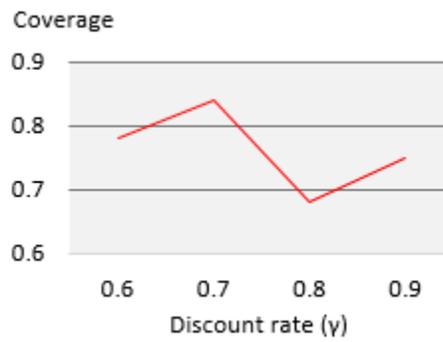
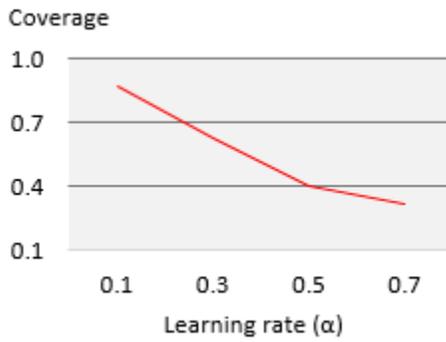
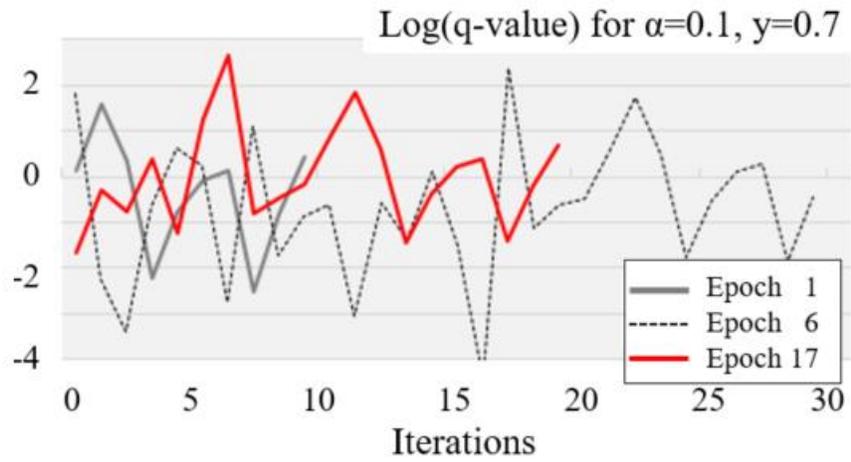


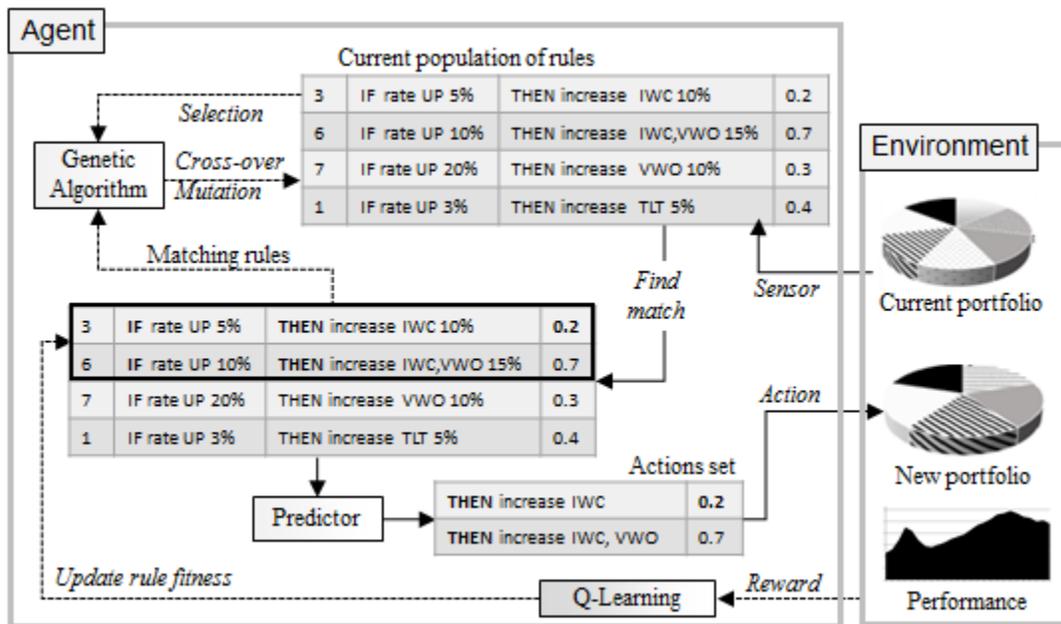
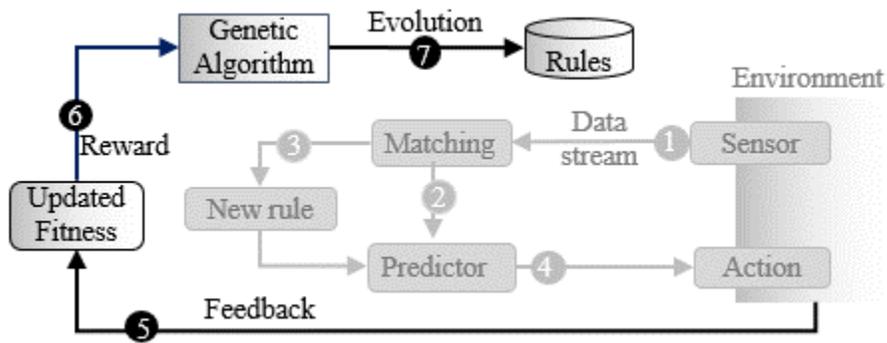
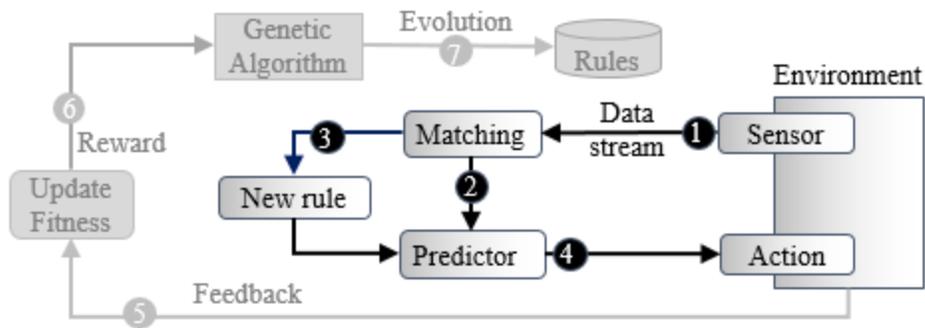
Q Action-Value map episode 1

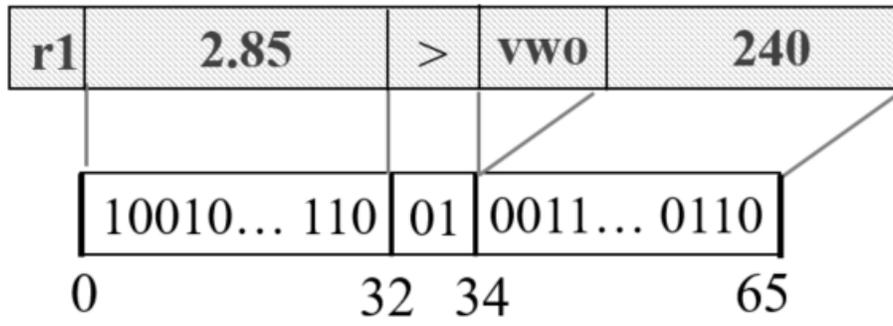


Q Action-Value map last episode







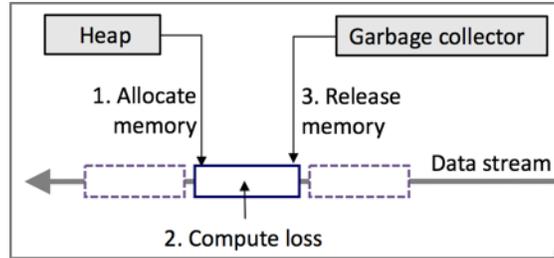


Chapter 16: Parallelism in Scala and Akka

Spark	Partitioner, Accumulator: <i>org.apache.spark</i> Broadcast: <i>org.apache.spark.broadcast</i> Resilient datasets: <i>org.apache.spark.rdd._</i> Data frame: <i>org.apache.spark.sql._</i> Caching, Shuffling: <i>org.apache.spark._</i> Listeners: <i>org.apache.spark.scheduler._</i> Serialization: <i>org.apache.spark.serializer</i>
Akka	Actors, Supervisors: <i>akka.actors._</i> Remote actors: <i>akka.remote</i> Type actors: <i>akka.actors._</i> Mailbox management: <i>akka.mailbox._</i> Clusters: <i>akka.cluster._</i> Dispatchers: <i>akka.dispatch</i> Events management: <i>akka.event._</i> Routing, Broadcast: <i>akka.routing</i> Persistency: <i>akka.persistence._</i>
Scala	Scheduler: <i>scala.actors.scheduler</i> Concurrency: <i>scala.concurrent</i> Par. collections: <i>scala.collection.parallel</i>
JVM	Threads, executors: <i>java.util.concurrent.*</i>

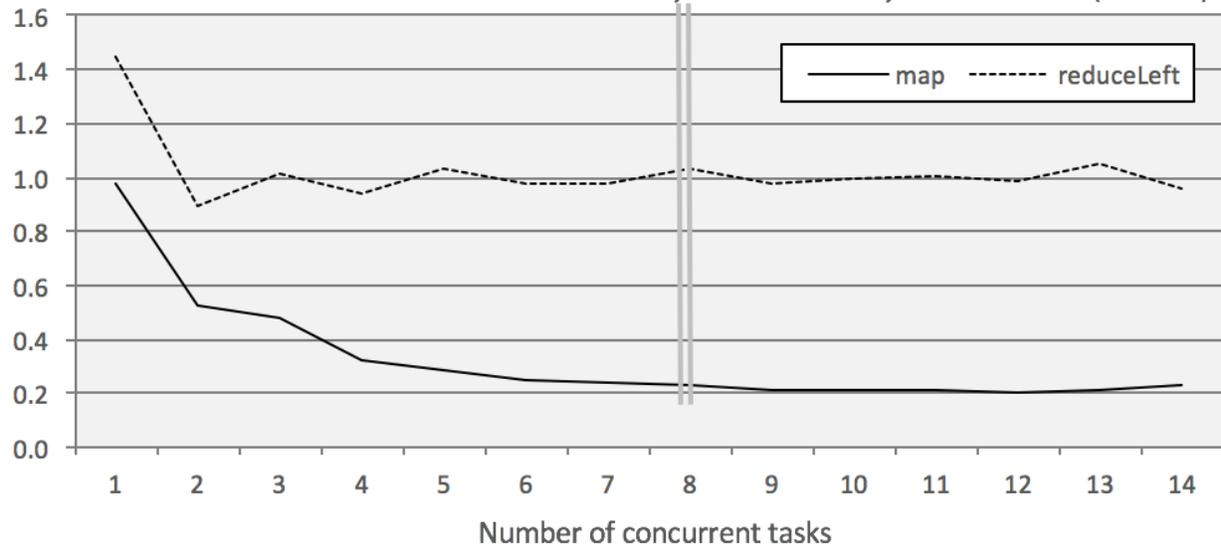
$$\mu_n = \frac{1}{n} \sum_{i=0}^{n-1} x_i$$

$$\mu_n = \left(1 - \frac{1}{n}\right)\mu_{n-1} + \frac{x_n}{n}$$



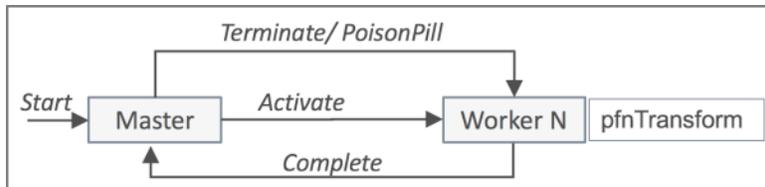
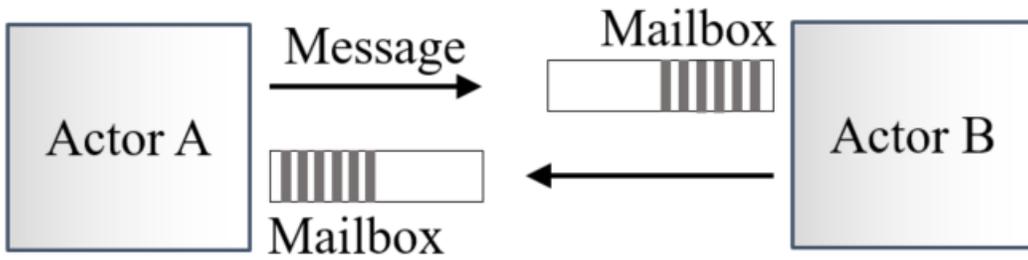
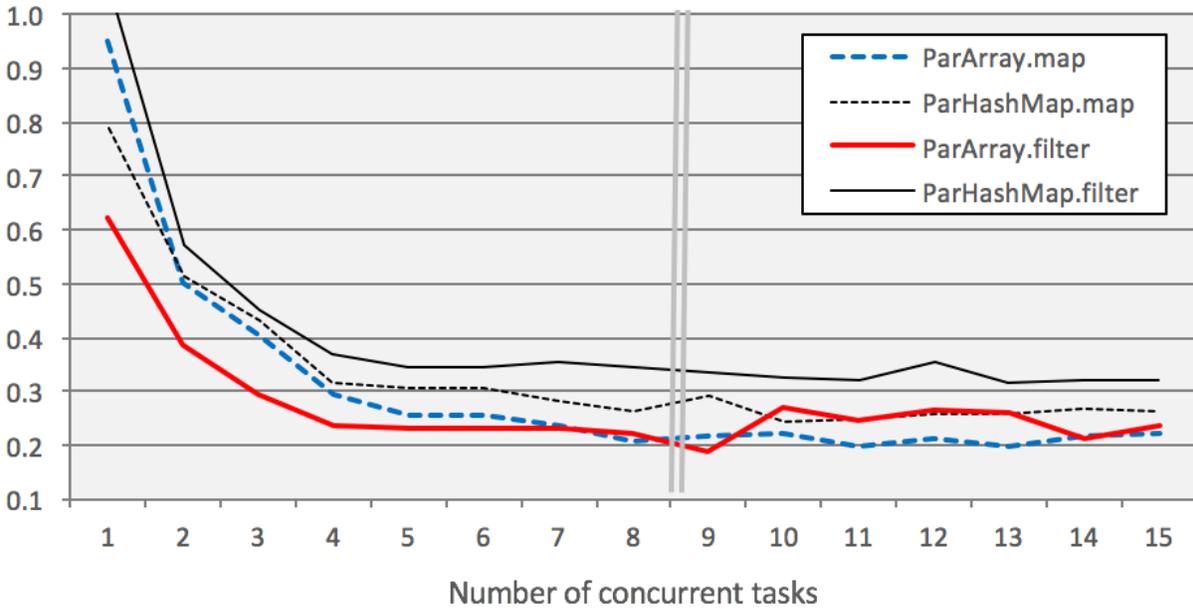
Execution time ratio

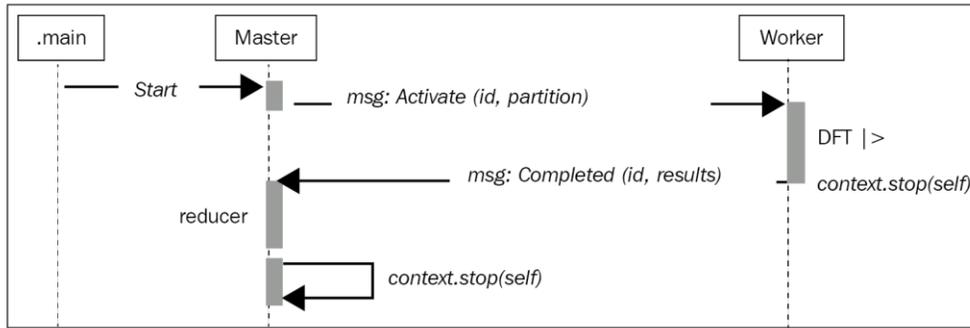
map and *reduce* methods execution time on *ParArray* relative to *Array* on 8 core CPU (-Xmx8G)



Execution time ratio

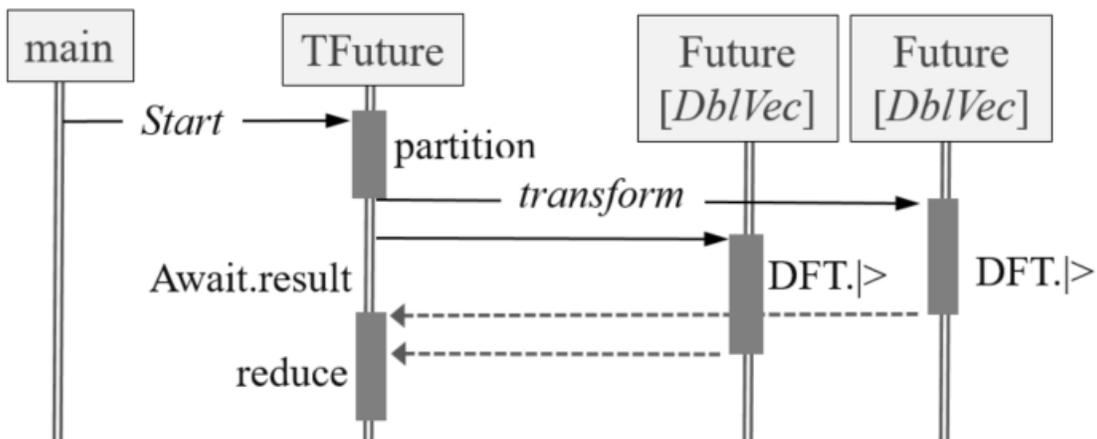
map and filter methods execution time on ParArray relative to Array and HashMap relative to ParHashMap on 8 core CPU (-Xmx8G)

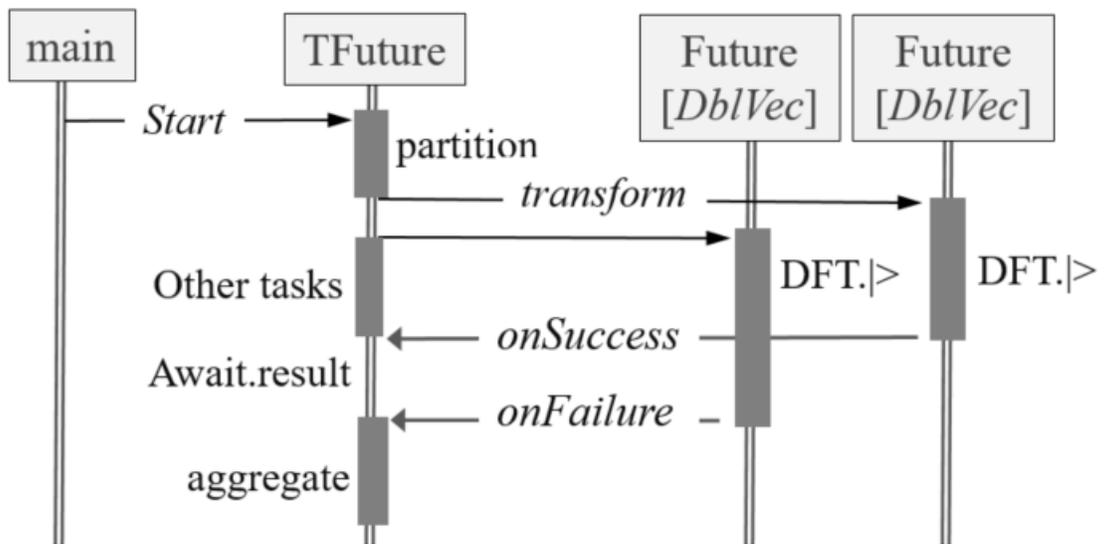




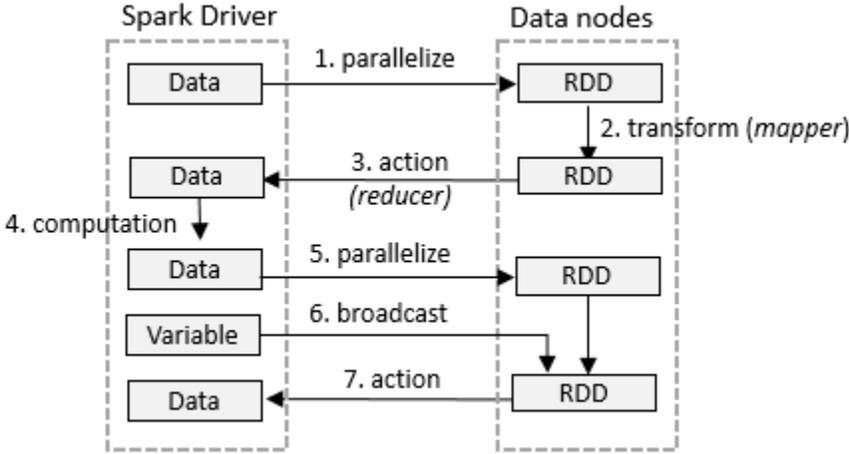
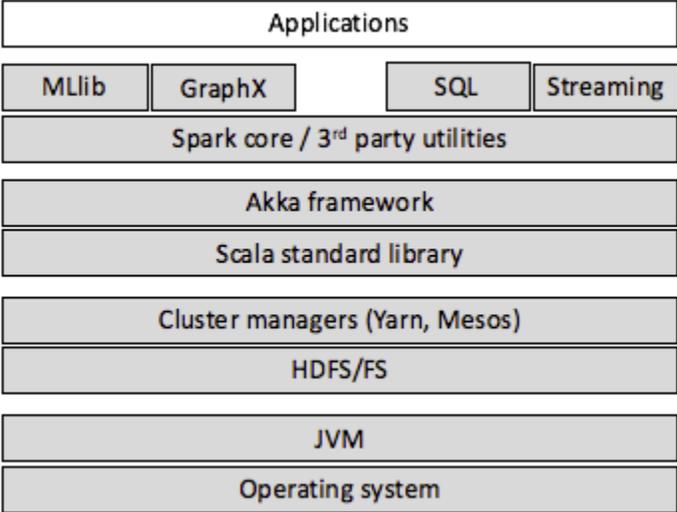
Execution time ratio

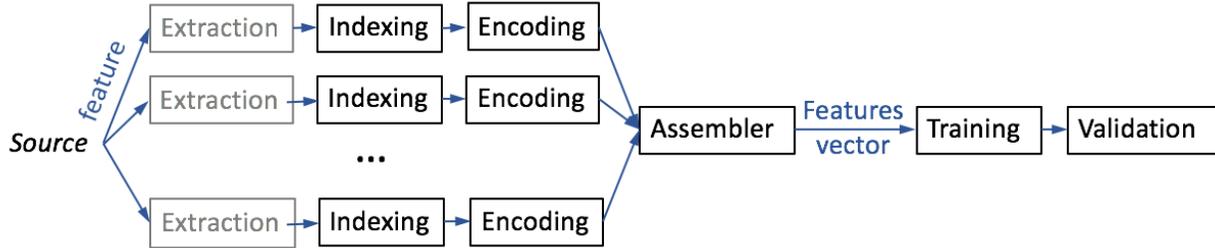
Execution time of the *Discrete Fourier transform* using *Akka master-workers* relative to the single threaded version





Chapter 17: Apache Spark MLlib





root

```

|-- date: string (nullable = true)
|-- asset: string (nullable = true)
|-- region: integer (nullable = true)
|-- agent: string (nullable = true)
|-- dateIndex: double (nullable = true)
|-- assetIndex: double (nullable = true)
|-- regionIndex: double (nullable = true)
|-- agentIndex: double (nullable = true)
|-- dateVector: vector (nullable = true)
|-- assetVector: vector (nullable = true)
|-- regionVector: vector (nullable = true)
|-- agentVector: vector (nullable = true)
|-- features: vector (nullable = true)
|-- rawPrediction: vector (nullable = true)
|-- probability: vector (nullable = true)
|-- prediction: double (nullable = true)

```

date	asset	region	agent
07/05/2014	300ec90b	27	aa5
08/03/2014	23c9ab02	7	a08

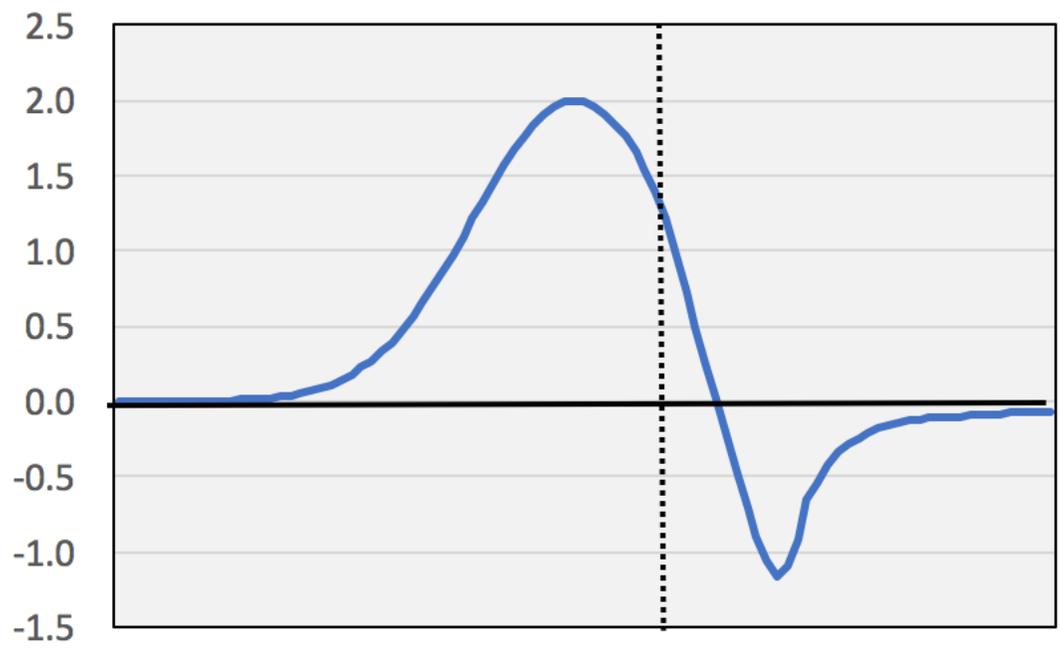
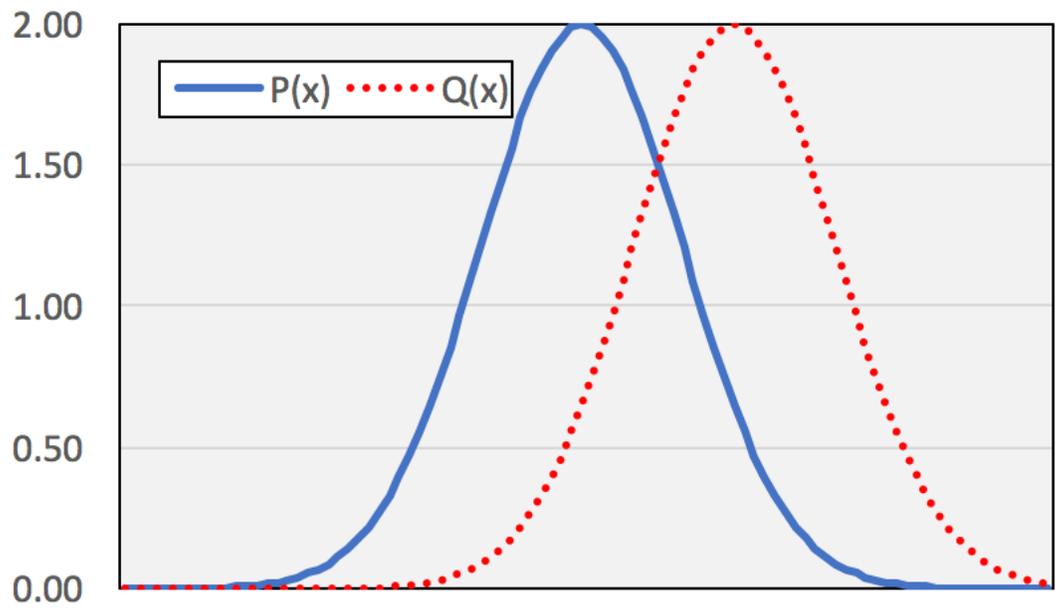
dateIndex	assetIndex	regionIndex	agentIndex
4.0	4.0	19.0	6.0
3.0	1.0	13.0	2.0

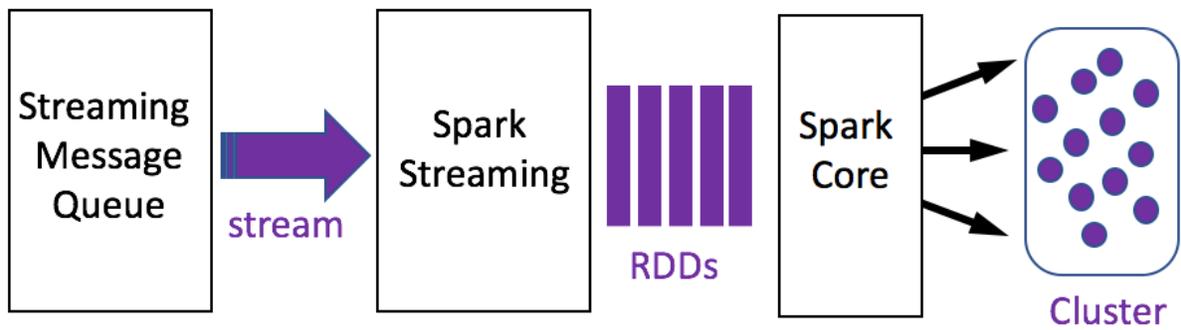
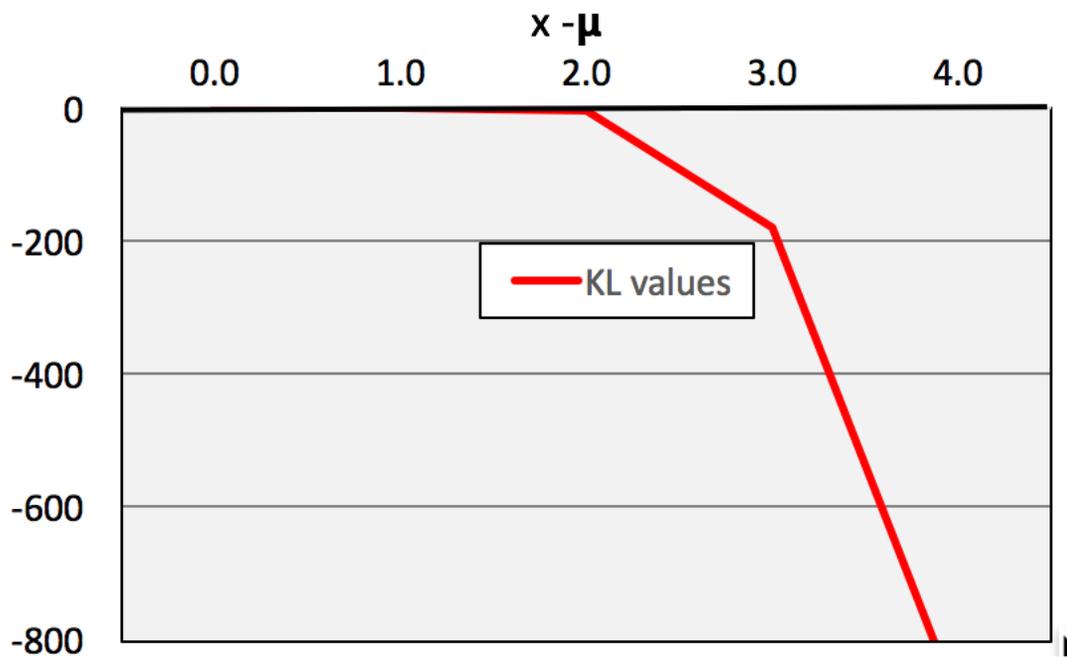
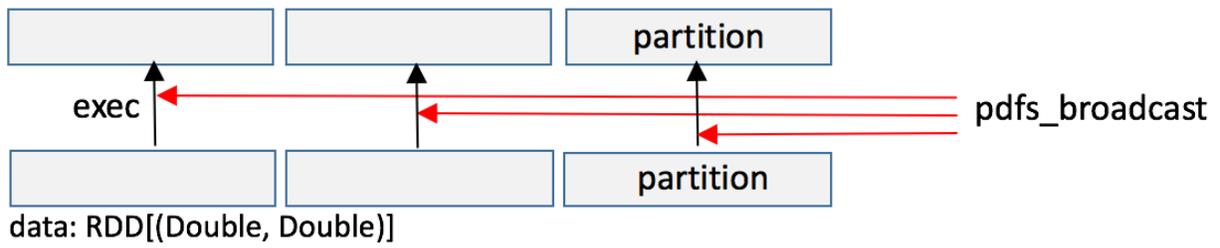
dateVector	assetVector	regionVector	agentVector	features
(16, [4], [1.0])	(14, [4], [1.0])	(20, [19], [1.0])	(15, [6], [1.0])	(65, [4, 20, 49, 56], ...)
(16, [3], [1.0])	(14, [1], [1.0])	(20, [13], [1.0])	(15, [2], [1.0])	(65, [3, 17, 43, 52], ...)

$$TPR = \frac{TP}{TP + FN} \quad TFP = \frac{TP}{TP + FP}$$

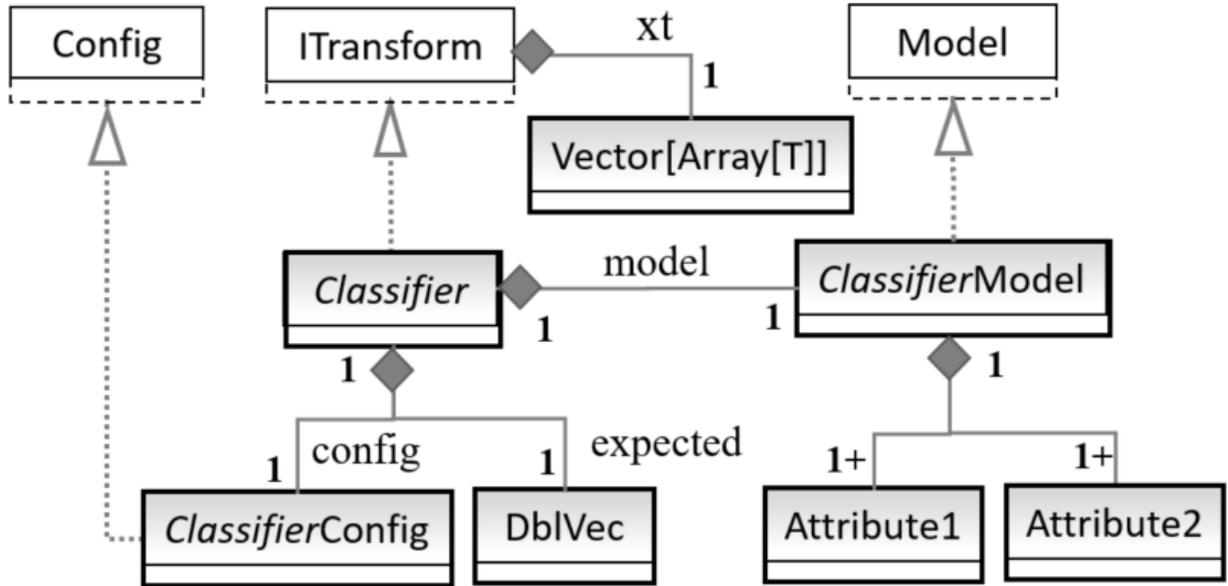
$$D_{KL}(P \parallel Q) = - \int_{-\infty}^{+\infty} p(x) \cdot \log \frac{p(x)}{q(x)}$$

$$D_{KL}(P \parallel Q) \neq D_{KL}(Q \parallel P)$$





Appendix A: Basic Concepts



$$J(f) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \dots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \dots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

$$f(x, y) = x^2 y + e^{-y} \quad J(f) = [2xy, x^2 - e^{-y}] \quad H(f) = \begin{bmatrix} 2y & 2x \\ 2x & e^{-y} \end{bmatrix}$$

$$x_{(t+1)} = x_{(t)} - \gamma \nabla F(a)$$

$$Ax = b \rightarrow \sum_{i=0}^{n-1} \alpha_i p_i x^* = b; p_i \cdot p_j = 0$$

$$F(x) = \sum_{i=0}^{n-1} f_i(x), \quad x_{t+1} = x_t - \alpha \sum_{i=0}^{n-1} \nabla f_i(x)$$

$$F(x_t + \Delta x) - F(x_t) \approx F'(x) \cdot \Delta x + \frac{1}{2} F''(x_t) (\Delta x)^2 \rightarrow x_{t+1} = x_t - \frac{F'(x_t)}{F''(x_t)}$$

$$H_t p_t = -\nabla F(x_t), \quad x_{t+1} = x_t + \alpha_t p_t$$

$$x_{t+1} = x_t + \Delta x_t; \quad \nabla F(x_t) + \Delta G_t = \Delta(\nabla F(x_t))$$

$$\mathcal{L}(w) = \sum_{i=0}^{m-1} r_i(w)^2; \quad r_i = y_i - F(x_i, w)$$

$$w_{(t+1)} = w_{(t)} - \left\| \frac{\partial r_i(w_{(t)})}{\partial w_i} \right\|_{ij}^{-1} r(w_{(t)})$$

$$\mathcal{L}(w + \delta) \approx \sum_{i=0}^{m-1} \left(r_i(w) - \frac{\partial F(x_i, w)}{\partial w} \delta \right)^2$$

$$\mathcal{L}(x, \lambda) = f(x) + \lambda(g(x) - c)$$

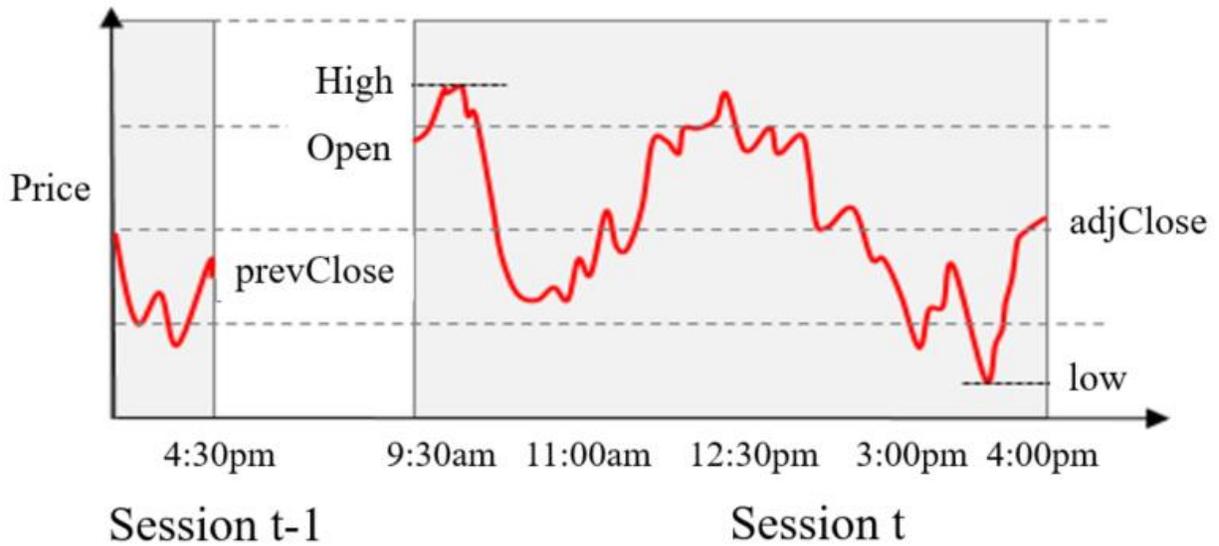
$$\nabla_{x, \lambda} \mathcal{L}(x, y) = 0$$

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial x_i} \\ \frac{\partial \mathcal{L}}{\partial \lambda} \end{bmatrix}$$

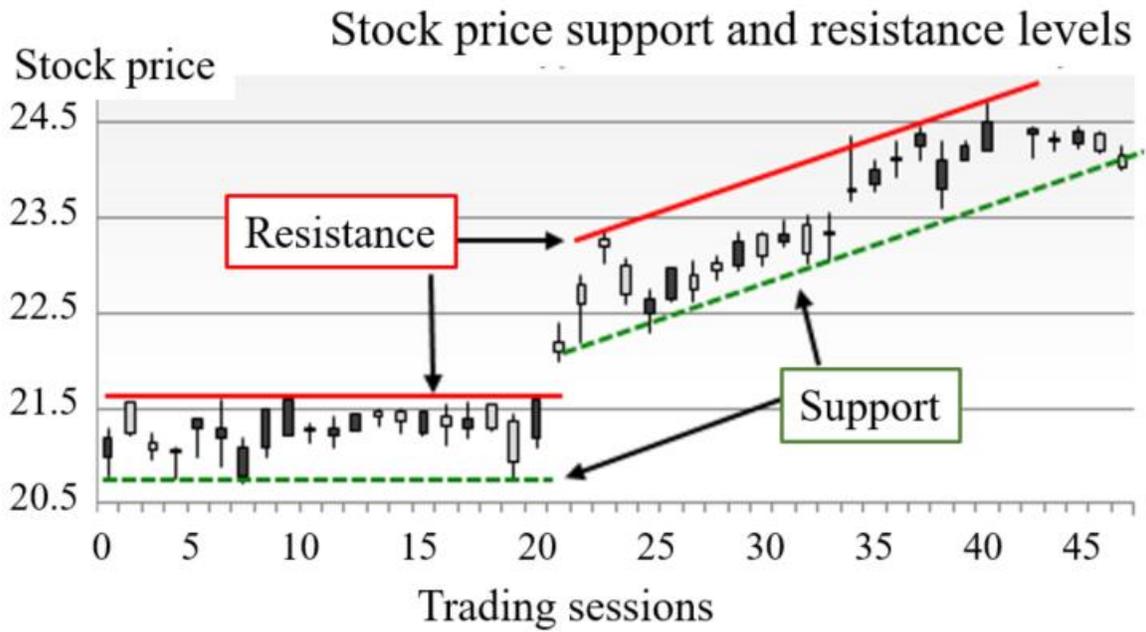
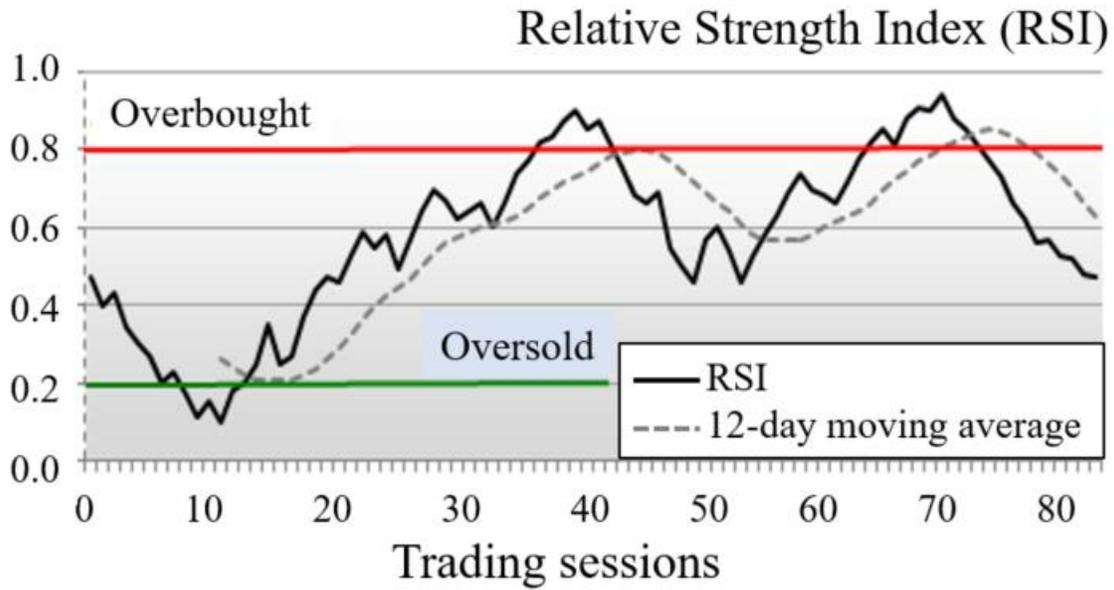
$$f(x, y) = x^2 + y^2 \text{ subject } x - y = 2$$

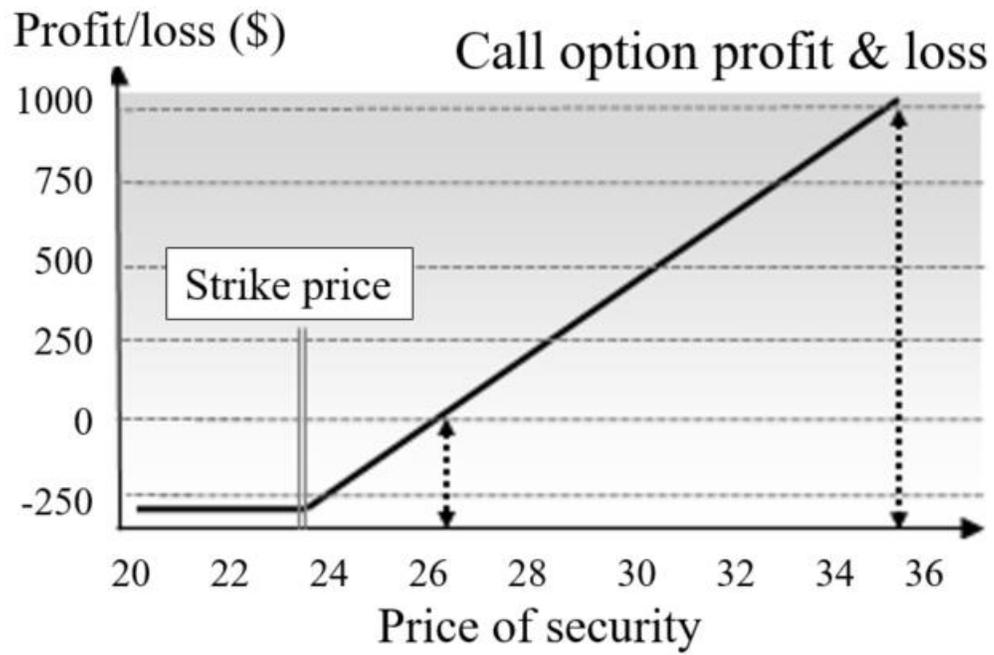
$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda, \frac{\partial \mathcal{L}}{\partial y} = 2y - \lambda, \frac{\partial \mathcal{L}}{\partial \lambda} = x - y - 2$$

$$x = 1, y = -1, \lambda = -2$$



$$U = p(t) - p(t-1), D = 0$$





▽